Taxonomy of Research in Digital Design

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Outline

- Research in Digital Design
  - Logic design as Engineering Science
  - Engineering thinking
    - Synthetic vs. Analysis
    - Algorithmic problem solving. Heuristics
    - Math problem solving
    - Complexity. Shannon Complexity
- Taxonomy of digital design
  - Root studies
    - Functional classification. Post theorem
    - NPN classification
    - Spectral transform
  - Body studies
    - Functional Decomposition
    - Spectral decomposition
    - Threshold logic
  - Branch and Leaves
    - Nano PLA
- Conclusions

Root problems

Belong to the pure math.
Form the Digital Design as an academic discipline

Root problems

Example 1

Functional Classification. Post Theorem

Universal Set of Logic Functions

Definition: Let $F = \{f_1, f_2, ..., f_m\}$ be a set of logic functions. If an arbitrary logic function may be realized by a loop-free combinational network using the logic elements that realize function $f_i \left(i = 1, 2, ..., m\right)$, then F is universal.

Definition: A function such that $f\left(0,0,...,0\right) = 0$ is a 0 - *preserving* function.
A function such that $f\left(1,1,...,1\right) = 1$ is a 1-preserving function.
Theorem : Let
M_0 be the set of 0-preserving functions,
M_1 be the set of 1-preserving functions,
M_2 be the set of self-dual functions,
M_3 be the set of isotonic functions, and
M_4 be the set of linear functions.
Then, the set of functions F is universal if $F \notin M_i \left(i = 0,1,2,3,4,5\right)$.

Root problems

Example 2

NPN Classification

NPN-EQUIVALENCE

For given logic function *f*, if a function *g* is derived from *f* by the combination of the following three operations:

1. Negation of some variables in *f*;
2. Permutation of some variables of *f*;
3. Negation of *f*,

then the functions *f* and *g* are NPN equivalent.

NPN-EQUIVALENCE

NPN transformation is one function transformation into another by Negations and Permutation of the variables and Negation of the function.

NPN class of the functions is a subset of function transformed into each other by Negations and Permutation of the variables and Negation of the function.

Classification of two-variable functions

# of variables	All functions	P	NP	NPN
0	0	0	0	0
	1	1	1	
1	x , y	x	x	x
	$\overline{x}$, $\overline{y}$	$\overline{x}$		
	xy	xy	xy	xy
	$\overline{x}\ y$	$\overline{x}\ y$		
	$x\ \overline{y}$			
	$\overline{x}\ \overline{y}$	$\overline{x}\ \overline{y}$		
2	$x + y$	$x + y$	$x + y$	
	$\overline{x} + y$	$\overline{x} + y$		
	$x + \overline{y}$			
	$\overline{x} + \overline{y}$	$\overline{x} + \overline{y}$		
	$x \oplus y$	$x \oplus y$	$x \oplus y$	$x \oplus y$
	$\overline{x} \oplus y$	$\overline{x} \oplus y$		

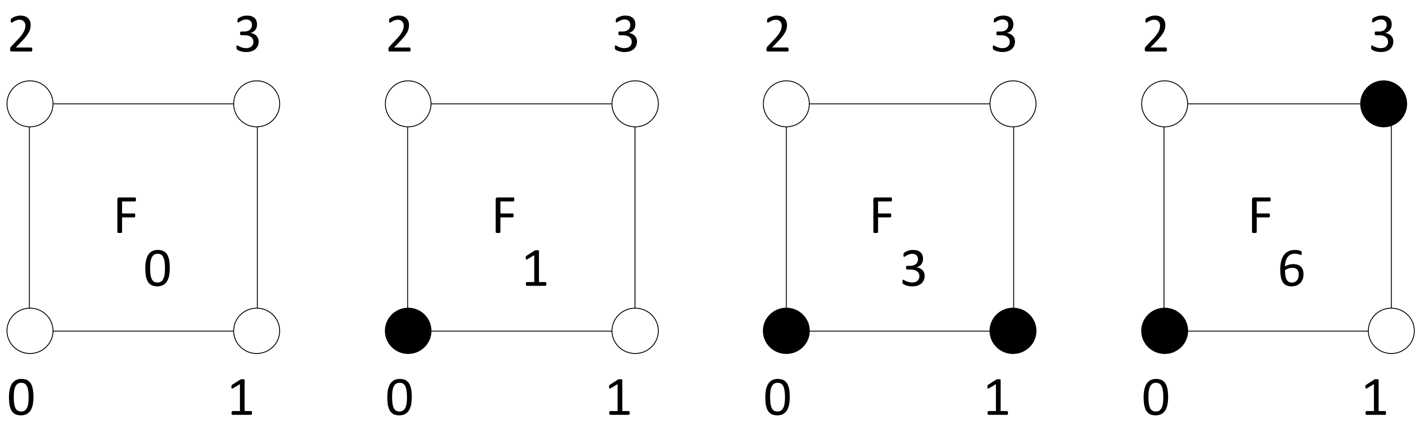
Number of equivalence classes

Number of variables	0	1	2	3	4
All functions	**2**	**4**	**16**	**256**	**65536**
P-equivalence	**2**	**4**	**12**	**80**	**3984**
NP-equivalence	**2**	**3**	**6**	**22**	**402**
NPN-equivalence	**1**	**2**	**4**	**14**	**222**

Example

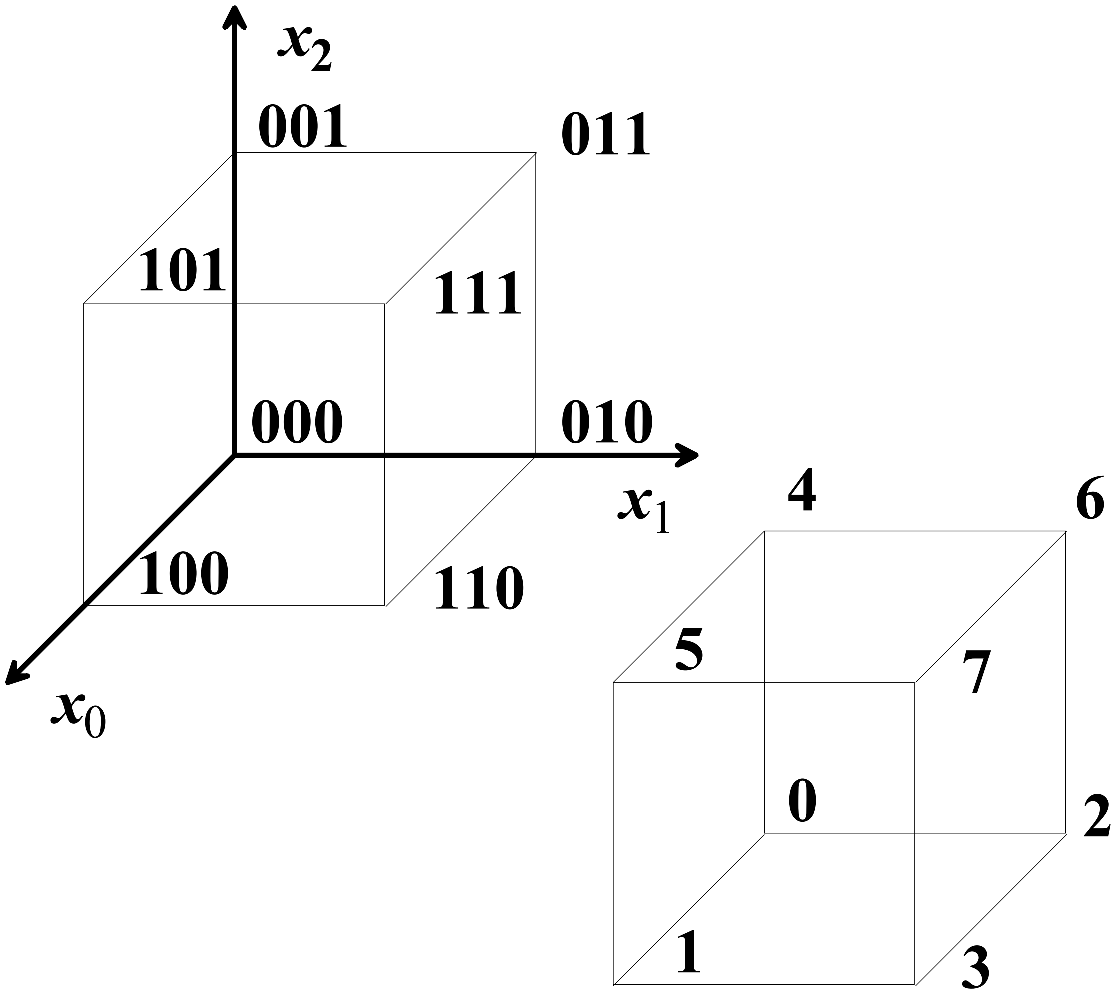
$$F_1(X_0, X_1) = \overline{F_{14}(X_0, X_1)} = F_2(\bar{X}_0, X_1) = \overline{F_{13}(\bar{X}_0, X_1)} =$$

$$= F_4(X_0, \bar{X}_1) = \overline{F_{11}(X_0, \bar{X}_1)} = F_8(\bar{X}_0, \bar{X}_1) = \overline{F_7(\bar{X}_0, \bar{X}_1)}$$

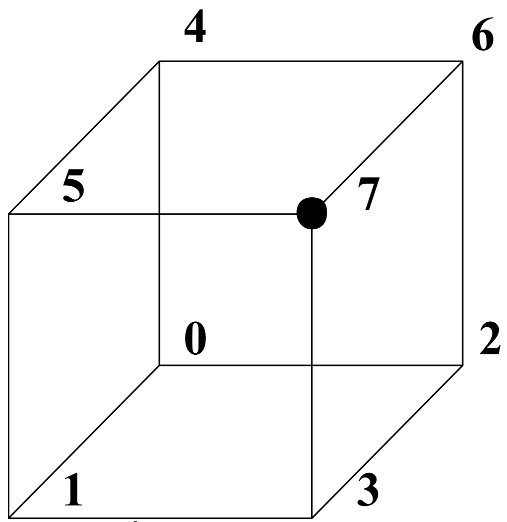


Functions of 2 variables have
4 NPN classes of equivalence.

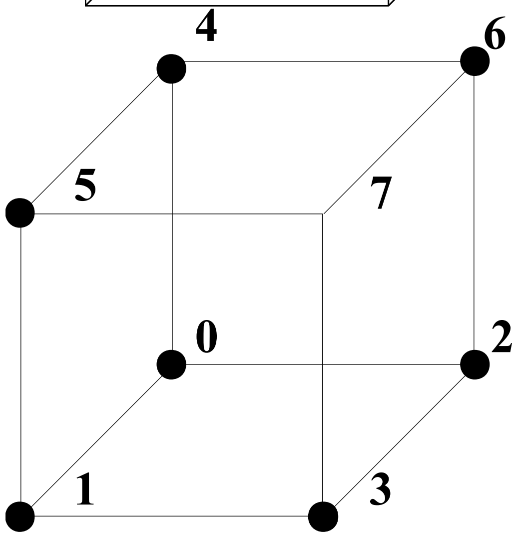
NPN classes of the functions of 3 variables



C_1 $N(C_1)=16$

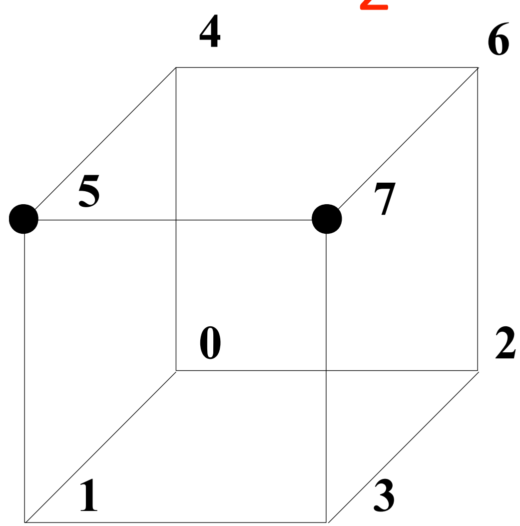


$$F(X) = X^7 = x_0 x_1 x_2$$

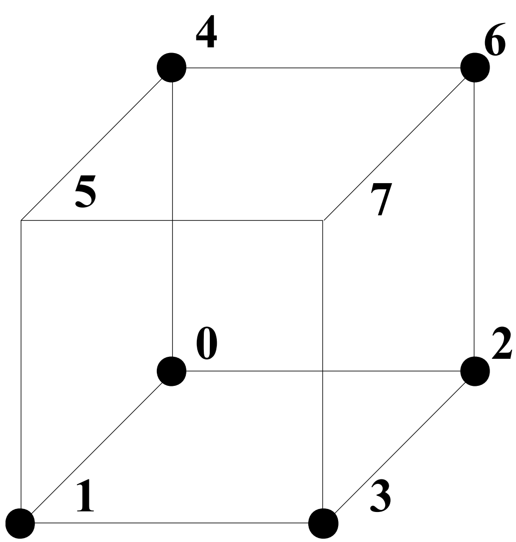


$$\overline{F(X)} = \overline{X^7} = \bar{x}_0 + \bar{x}_1 + \bar{x}_2$$

C_2 $N(C_2)=24$

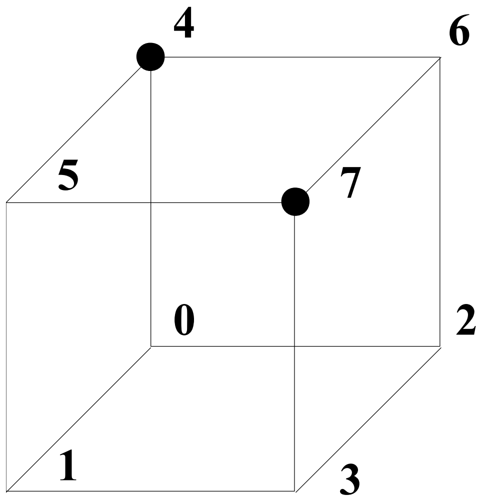


$$F(X) = X^7 + X^5 = x_0 x_1 x_2 + x_0 \bar{x}_1 x_2$$

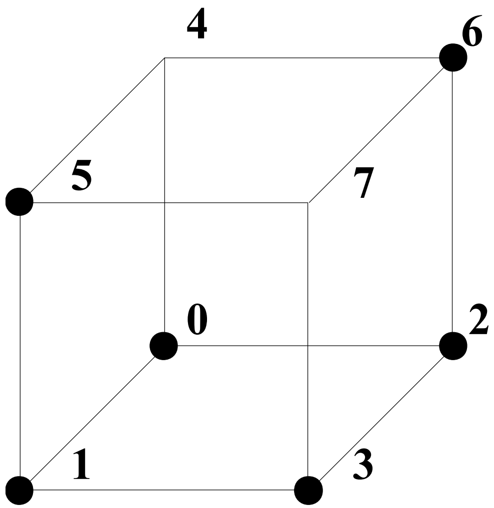


$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^5} = (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(\bar{x}_0 + x_1 + \bar{x}_2)$$

C_3 $\quad N(C_3)=24$

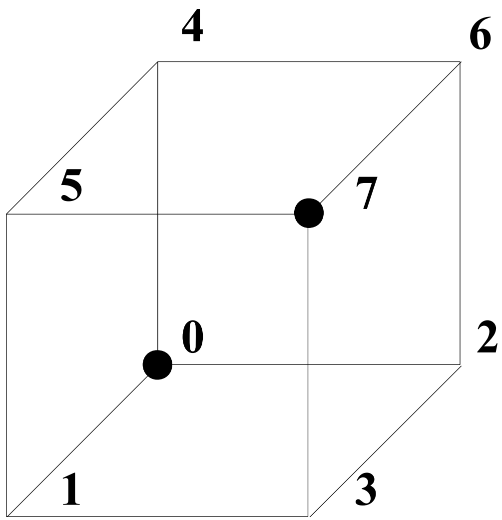


$$F(X) = X^7 + X^4 = x_0 x_1 x_2 + \bar{x}_0 \bar{x}_1 x_2$$

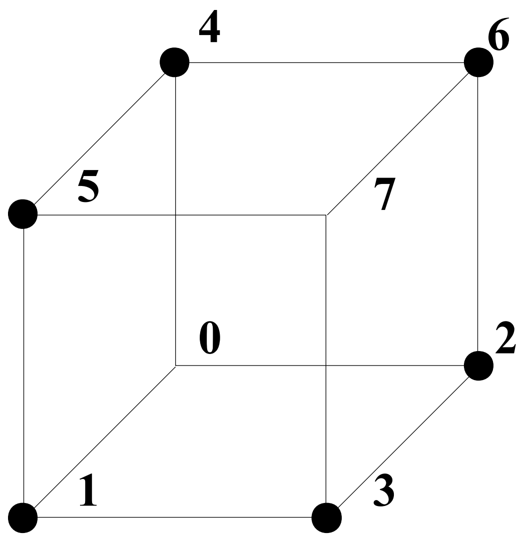


$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^4} = (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(x_0 + x_1 + \bar{x}_2)$$

C_4 $\quad N(C_4)=8$



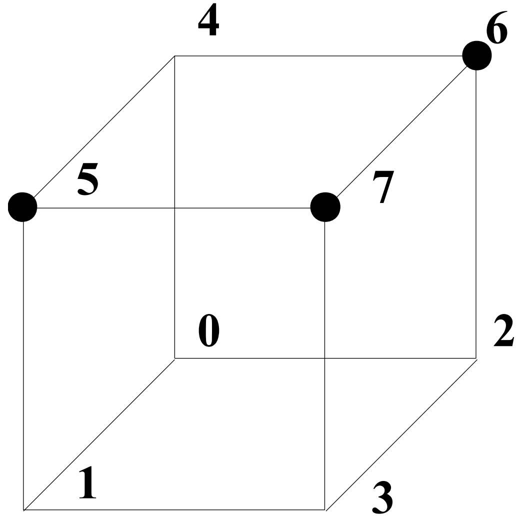
$$F(X) = X^7 + X^0 = x_0x_1x_2 + \bar{x}_0\bar{x}_1\bar{x}_2$$

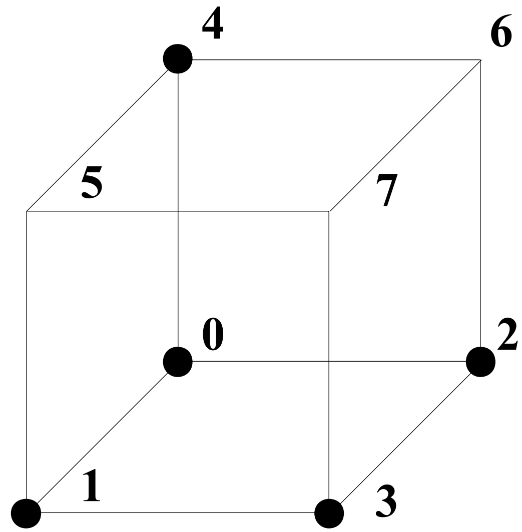


$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^0} =$$

$$= (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(x_0 + x_1 + x_2)$$

C_5 $N(C_5)=48$



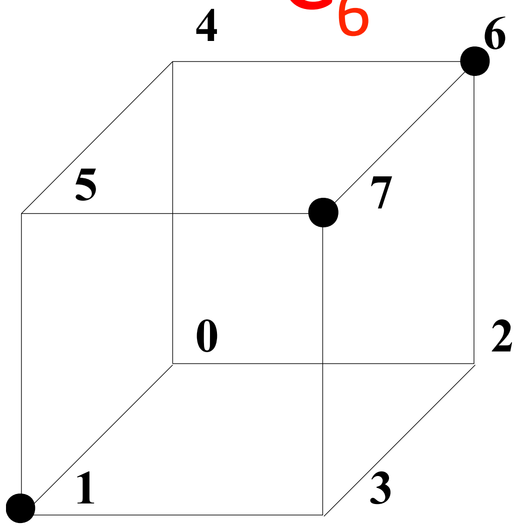


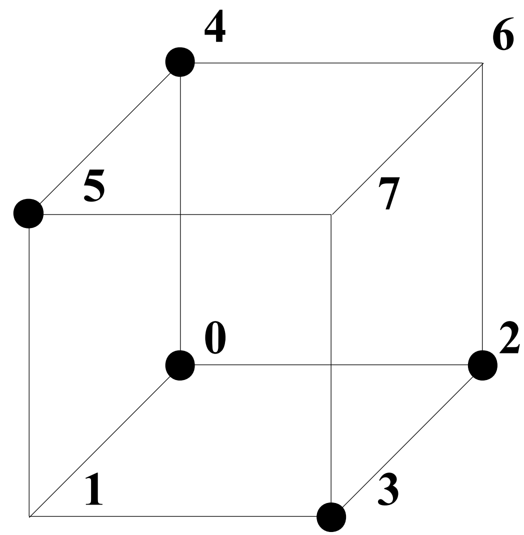
$$F(X) = X^7 + X^6 + X^5 = x_0 x_1 x_2 + \bar{x}_0 x_1 x_2 + x_0 \bar{x}_1 x_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^6} \bullet \overline{X^5} =$$

$$= (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(x_0 + \bar{x}_1 + \bar{x}_2)(\bar{x}_0 + x_1 + \bar{x}_2)$$

C_6 $\qquad$ $N(C_6)=48$



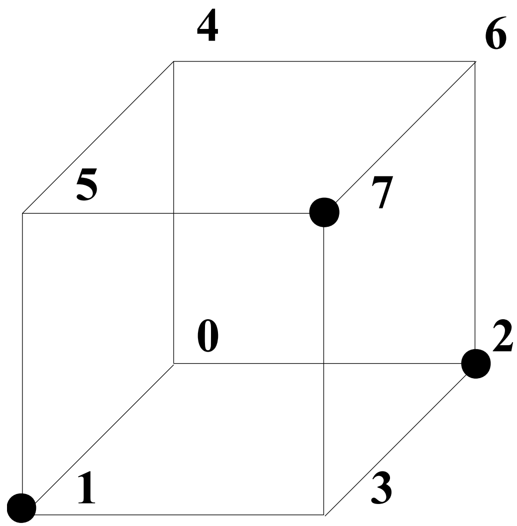


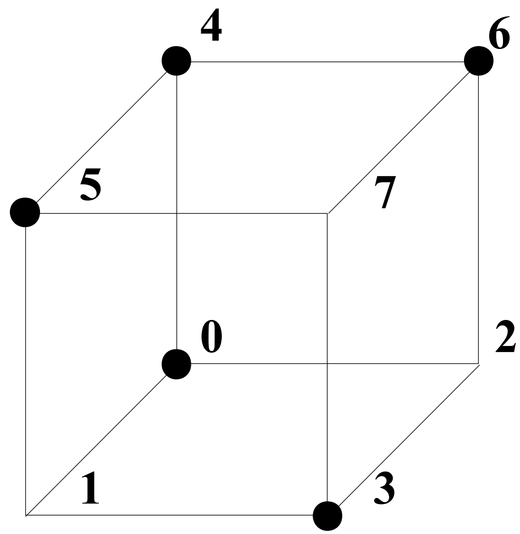
$$F(X) = X^7 + X^6 + X^1 = x_0x_1x_2 + \bar{x}_0x_1x_2 + x_0\bar{x}_1\bar{x}_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^6} \bullet \overline{X^1} =$$

$$= (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(x_0 + \bar{x}_1 + \bar{x}_2)(\bar{x}_0 + x_1 + x_2)$$

C_7 $N(C_7)=16$



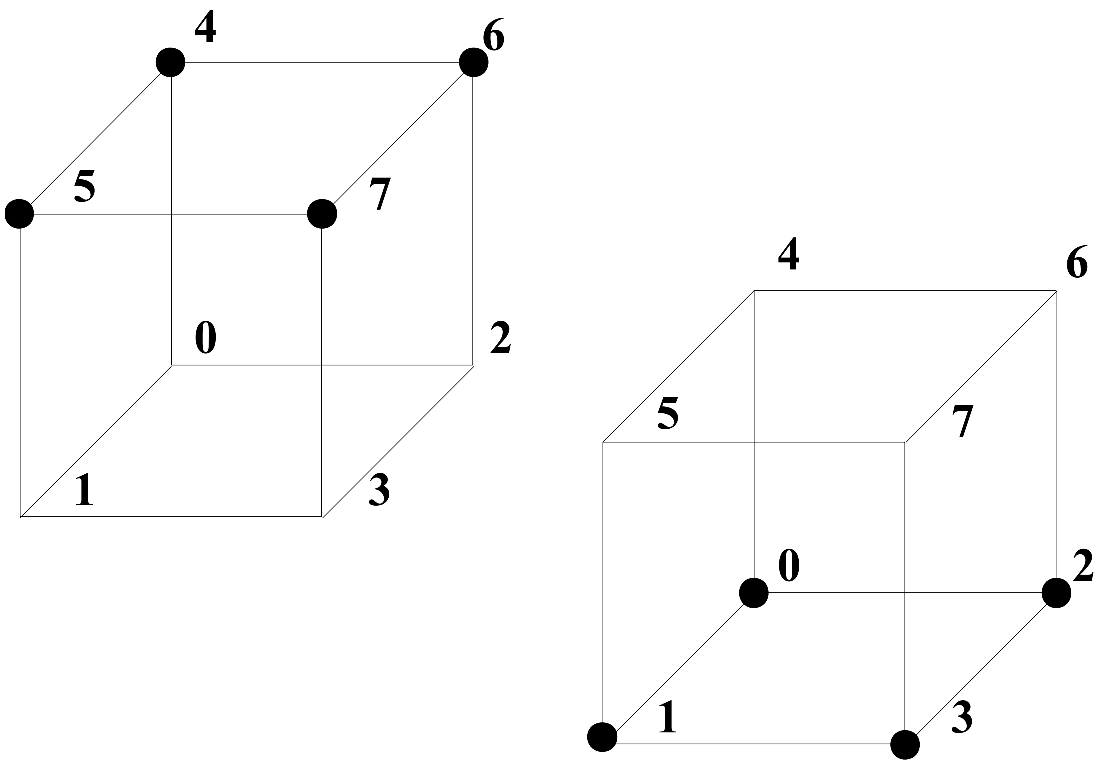


$$F(X) = X^7 + X^2 + X^1 = x_0 x_1 x_2 + \bar{x}_0 x_1 \bar{x}_2 + x_0 \bar{x}_1 \bar{x}_2$$

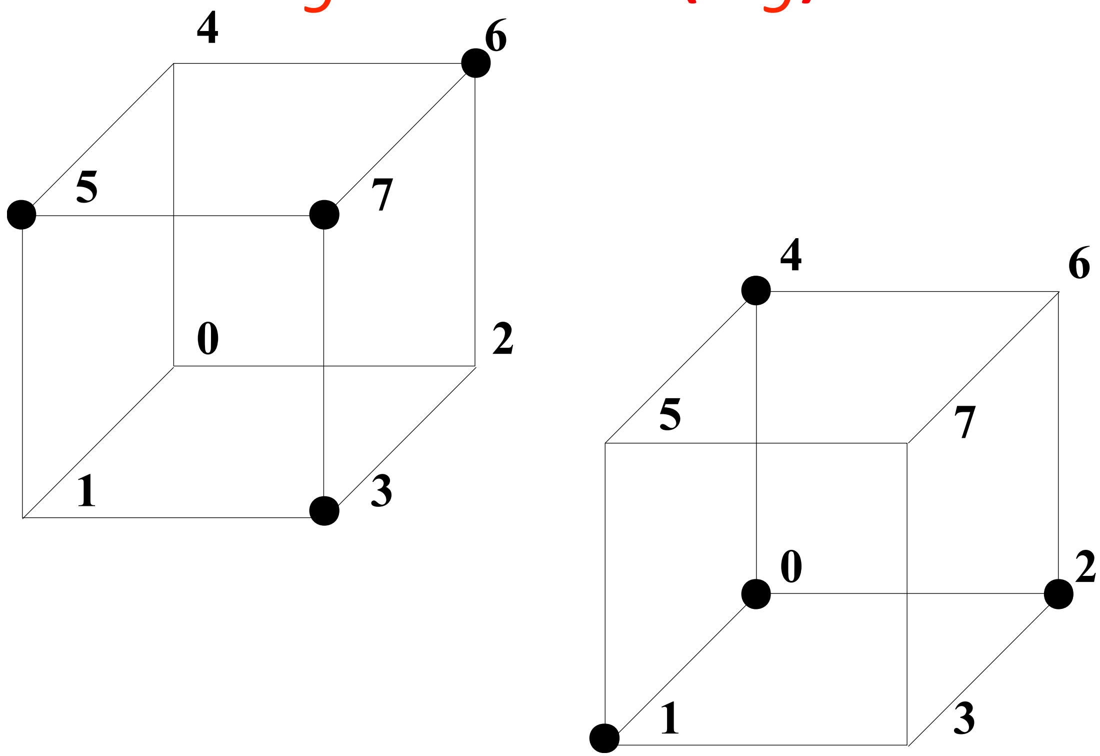
$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^2} \bullet \overline{X^1} =$$

$$= (\bar{x}_0 + \bar{x}_1 + \bar{x}_2)(x_0 + \bar{x}_1 + x_2)(\bar{x}_0 + x_1 + x_2)$$

C_8 $N(C_8)=6$



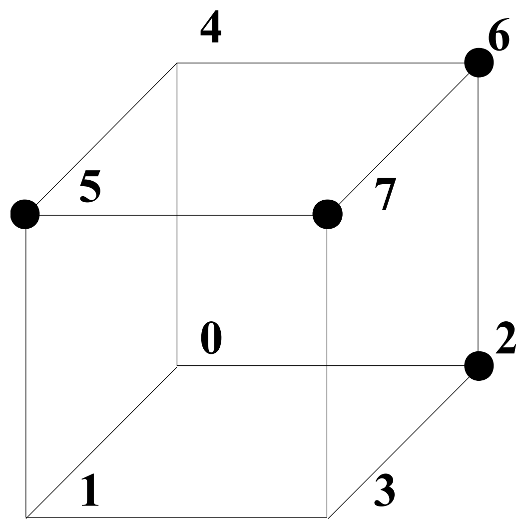
C_9 $N(C_9)=8$

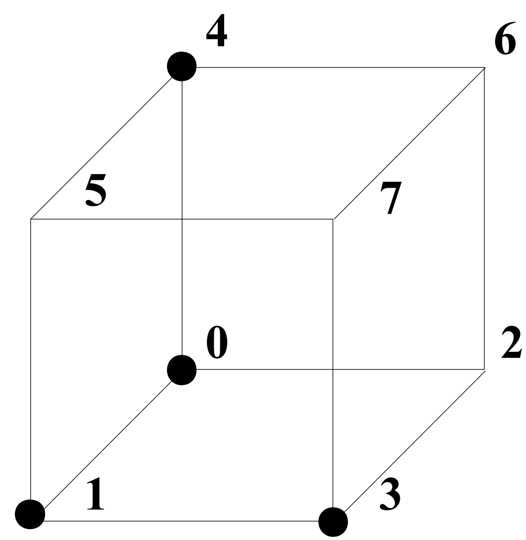


$$F(X) = X^7 + X^6 + X^5 + X^3 =$$
$$= x_0 x_1 x_2 + \bar{x}_0 x_1 x_2 + x_0 \bar{x}_1 x_2 + x_0 x_1 \bar{x}_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^6} \bullet \overline{X^5} \bullet \overline{X^1} = X^4 + X^2 + X^1 + X^0$$

C_{10} $N(C_{10})=24$



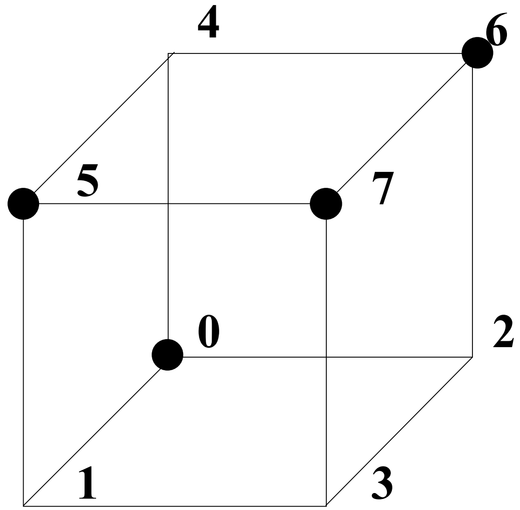


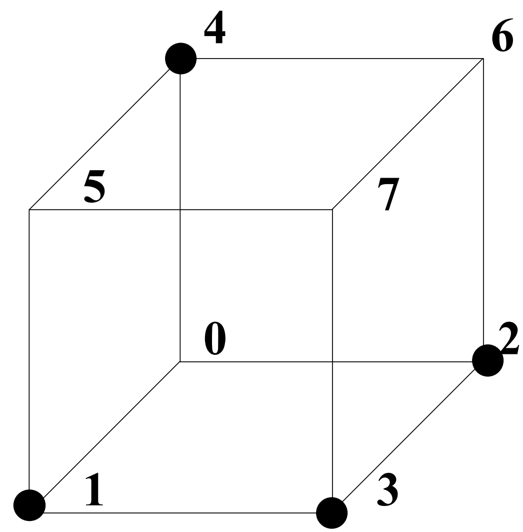
$$F(X) = X^7 + X^6 + X^5 + X^2 =$$

$$= x_0 x_1 x_2 + \bar{x}_0 x_1 x_2 + x_0 \bar{x}_1 x_2 + \bar{x}_0 x_1 \bar{x}_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^6} \bullet \overline{X^5} \bullet \overline{X^2} = X^4 + X^3 + X^1 + X^0$$

C_{11} $N(C_{11})=24$



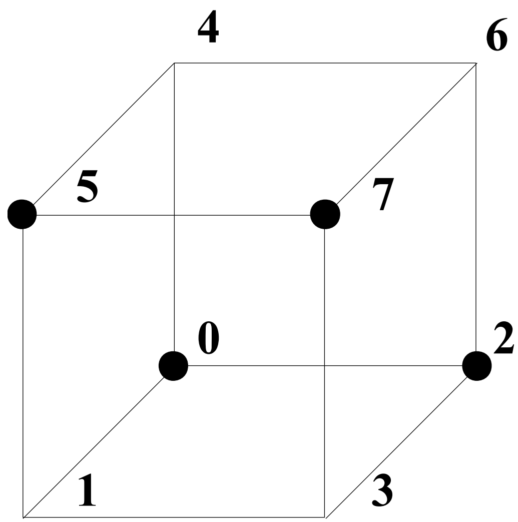


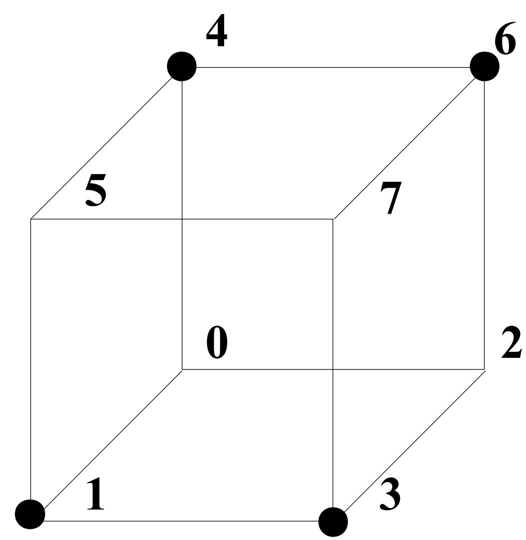
$$F(X) = X^7 + X^6 + X^5 + X^0 =$$

$$= x_0 x_1 x_2 + \bar{x}_0 x_1 x_2 + x_0 \bar{x}_1 x_2 + \bar{x}_0 \bar{x}_1 \bar{x}_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^6} \bullet \overline{X^5} \bullet \overline{X^0} = X^4 + X^3 + X^1 + X^2$$

C_{12} $N(C_{12})=6$



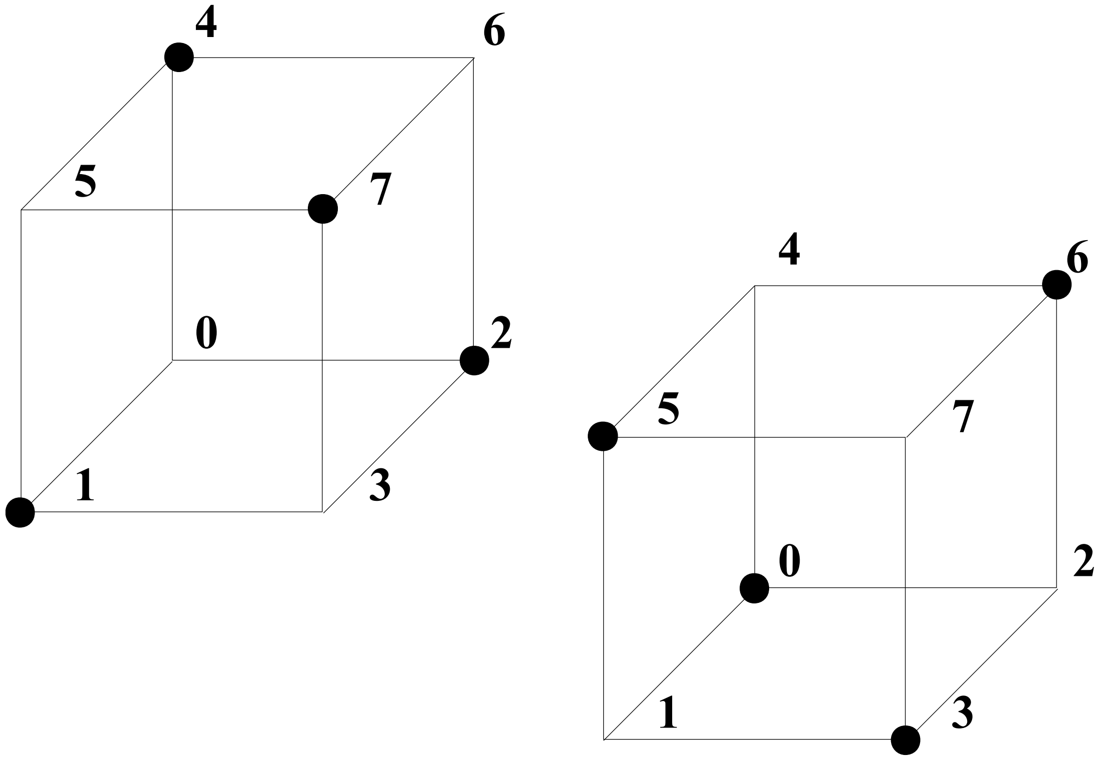


$$F(X) = X^7 + X^5 + X^2 + X^0 =$$

$$= x_0 x_1 x_2 + x_0 \bar{x}_1 x_2 + \bar{x}_0 x_1 \bar{x}_2 + \bar{x}_0 \bar{x}_1 \bar{x}_2$$

$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^5} \bullet \overline{X^2} \bullet \overline{X^0} = X^4 + X^6 + X^1 + X^3$$

C_{13} $\quad$ $N(C_{13})=2$

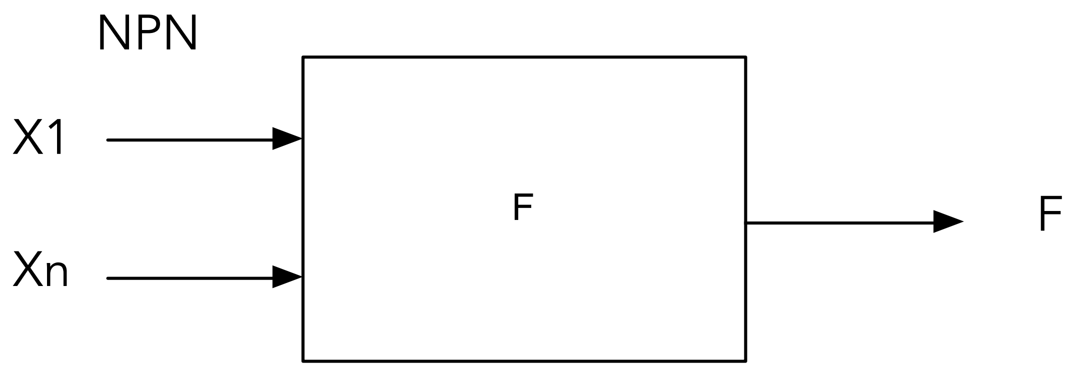


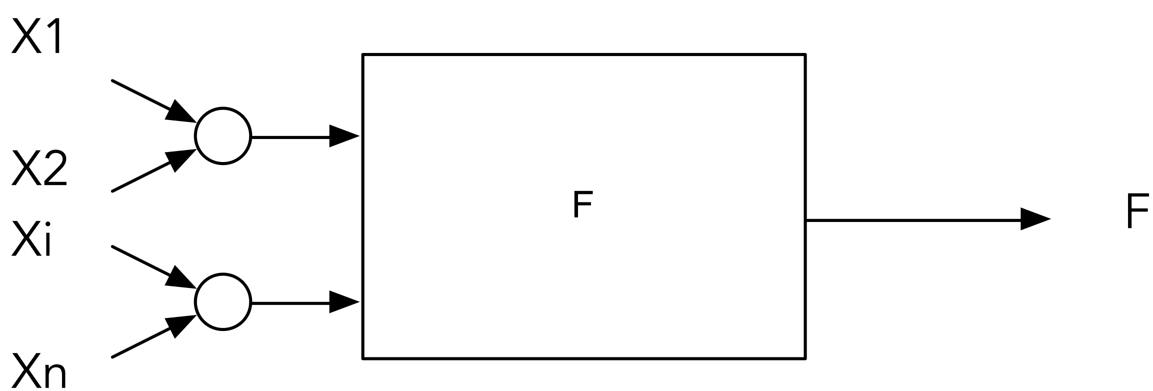
$$F(X) = X^7 + X^4 + X^2 + X^1 =$$

$$= x_0 x_1 x_2 + \bar{x}_0 \bar{x}_1 x_2 + \bar{x}_0 x_1 \bar{x}_2 + x_0 \bar{x}_1 \bar{x}_2$$

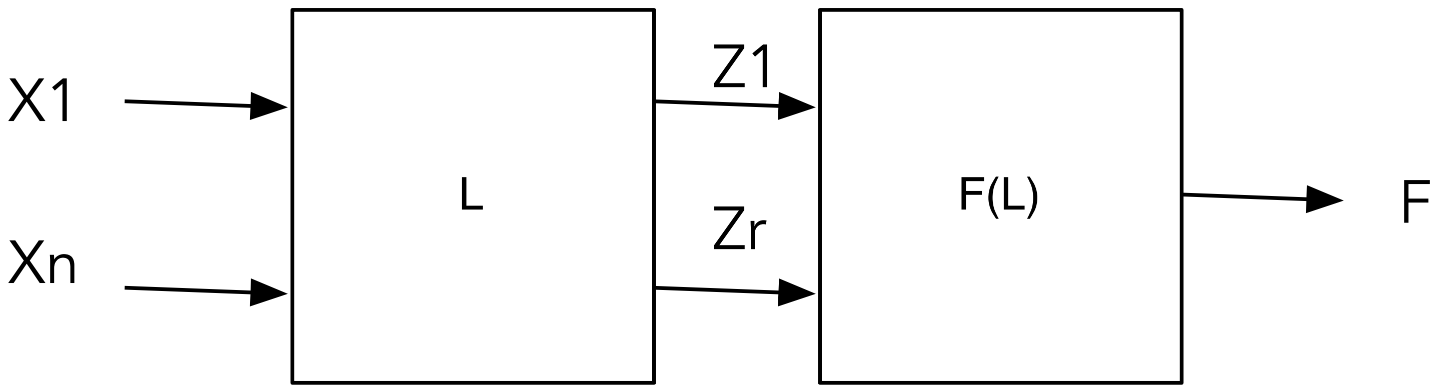
$$\overline{F(X)} = \overline{X^7} \bullet \overline{X^4} \bullet \overline{X^2} \bullet \overline{X^1} = X^6 + X^5 + X^3 + X^0$$

From NPN to Linear Decomposition





Linear Decomposition



The value of the autocorrelation function R_f of the logical function f: $GF(2^n) \Rightarrow GF(2)$ at the point $\tau \in$ GF(2^n) is defined as:

$$R_f(\tau) = \sum_{x \in GF(2^n)} f(x) f(x+\tau)$$

Body problems

Fundamental studies of the Digital Design itself, enriching the discipline

Body problems

Example 1

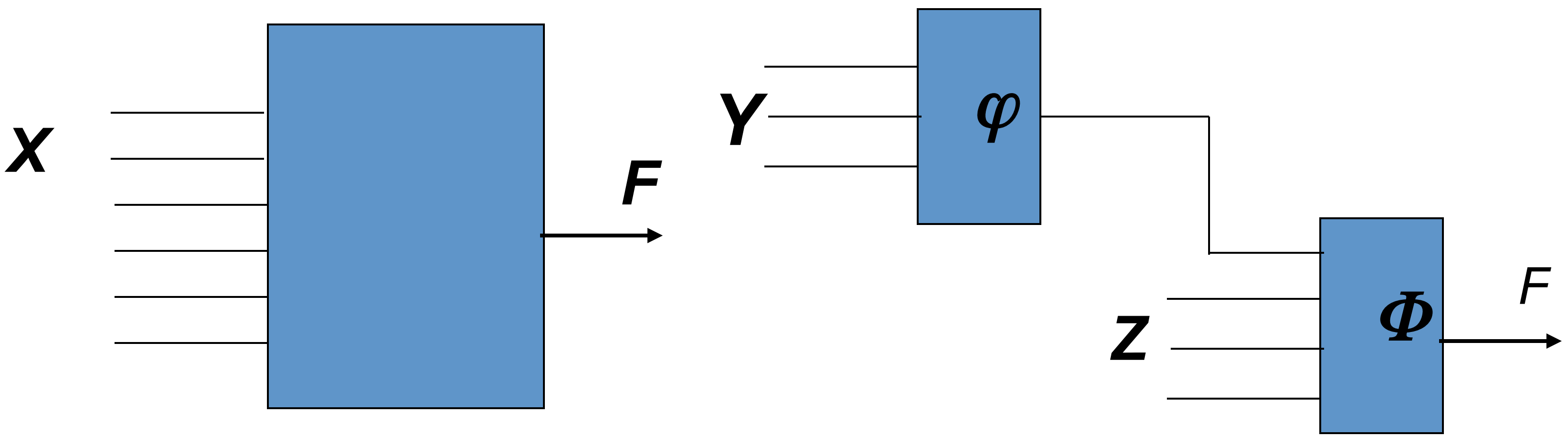
Functional Decomposition

Two-Level Decomposition

$$F(X) = F(Z,Y) \quad Z \cup Y = X, Z \cap Y = \varnothing$$

$$F(Z,Y) = \Phi(Z, \varphi(Y))$$

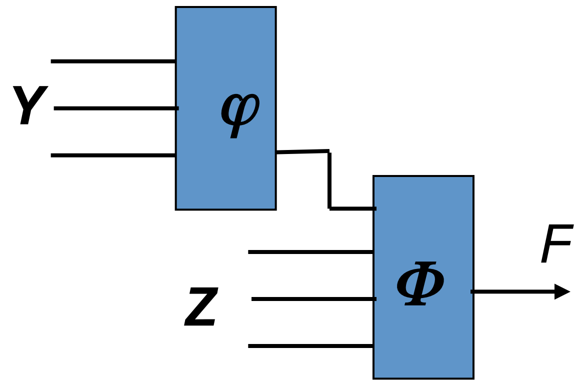
Y = bound set Z= free set



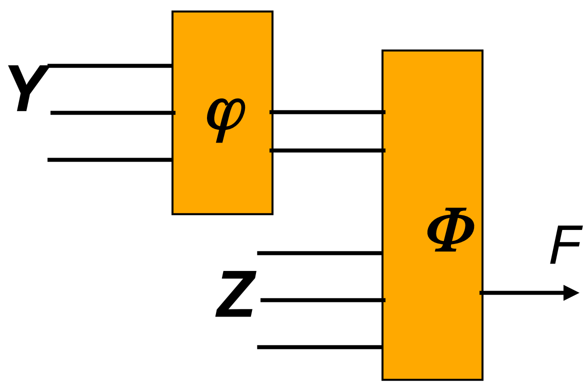
$If\ Z \cap Y = \varnothing$ - *disjoint decomposition*

$If\ Z \cap Y \neq \varnothing$ - *non-disjoint decomposition*

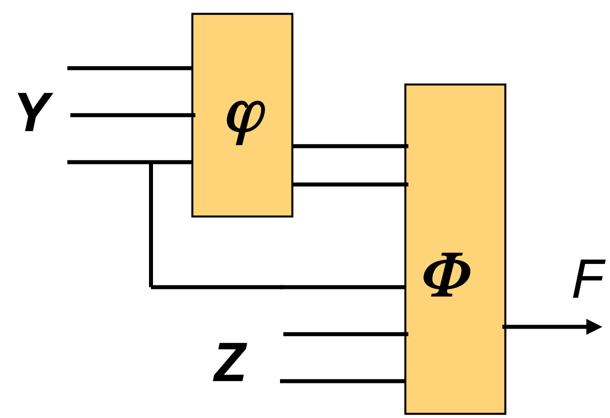
Decomposition Types



Simple disjoint decomposition
(Asenhurst)



Disjoint decomposition
(Curtis)



Non-disjoint decomposition

Simple Disjoint Decomposition

If the function

$$F(X) = F(Z,Y) \quad Z \cup Y = X, Z \cap Y = \varnothing$$

can be presented in a form

$$F(Z,Y) = \Phi(Z, \varphi(Y))$$

we will say that function $F(X)$ has

a simple disjoint decomposition with respect to bisection (Z,Y).

$\Phi(\Theta)$ *is an image of the decomposition*;

$\varphi(Y)$ *is a component of the decomposition.*

Example

$$F(X) = F(x_0, x_1, x_2, x_3, x_4) = \Phi(x_0, x_1, \varphi(x_2, x_3, x_4)) =$$

$$= \varphi\bar{x}_0\bar{x}_1 + \bar{\varphi}x_0x_1$$ - *image of the decomposition,*

$$\varphi = x_2x_3 + x_2x_4 + x_3x_4$$ - *component of the decomposition.*

$$F(X) = (x_2x_3 + x_2x_4 + x_3x_4)\bar{x}_0\bar{x}_1 +$$

$$+\overline{(x_2x_3 + x_2x_4 + x_3x_4)}x_0x_1 =$$

$$= \bar{x}_0\bar{x}_1x_2x_3 + \bar{x}_0\bar{x}_1x_2x_4 + \bar{x}_0\bar{x}_1x_3x_4 +$$

$$+ x_0x_1\bar{x}_2\bar{x}_3 + x_0x_1\bar{x}_2\bar{x}_4 + x_0x_1\bar{x}_3\bar{x}_4$$

with decomposition - 11 two-input logical operations;

without decomposition - 23 two-input logical operations.

Decomposition Chart

$$F(X) = F(Z,Y) \quad Z \cup Y = X,\ Z \cap Y = \varnothing$$

	Ω_Y	0	1	2	3	4	5	6	7
	Y	000	100	010	110	001	101	011	111
Ω_Z	Z								
0	00	0	0	0	1	0	1	1	1
1	10	0	0	0	0	0	0	0	0
2	01	0	0	0	0	0	0	0	0
3	11	1	1	1	0	1	0	0	0
		$\varphi(Y) = \varphi(x_2, x_3, x_4)$							
		0	0	0	1	0	1	1	1

Decomposition Chart

		$\varphi(Y)=$	$\varphi(Y)=$
Z		0	1
Ω_0	$00\ldots00$	$f_2(\Omega_0)$	$f_1(\Omega_0)$
Ω_1	$10\ldots00$	$f_2(\Omega_1)$	$f_1(\Omega_1)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\Omega_{2^{n-k}-1}$	$11\ldots11$	$f_2\left(\Omega_{2^{n-k}-1}\right)$	$f_1\left(\Omega_{2^{n-k}-1}\right)$

Example:

$$F(X) = \Phi\left(Z, \phi(Y)\right) =$$

$$= \phi f_1(Z) + \bar{\phi} f_2(Z) =$$

$$= \phi \bar{x}_0 \bar{x}_1 + \bar{\phi} x_0 x_1$$

$$f_1(Z) = \Phi(Z, \phi(Y) = 1) = \overline{x_0}\,\overline{x_1};$$

$$f_2(Z) = \Phi(Z, \phi(Y) = 0) = x_0 x_1.$$

$$F(X) = F(x_0, x_1, x_2, x_3, x_4) =$$
$$= \Phi(x_0, x_1, \underline{\phi}(x_2, x_3, x_4)) =$$
$$= \phi \bar{x}_0 \bar{x}_1 + \phi x_0 x_1$$

$$\phi = x_2 x_3 + x_2 x_4 + x_3 x_4$$

$$F(X) = (x_2 x_3 + x_2 x_4 + x_3 x_4) \bar{x}_0 \bar{x}_1 +$$
$$+ (x_2 x_3 + x_2 x_4 + x_3 x_4) x_0 x_1$$

Example

	Ω_Y	0	1	2	3	4	5	6	7
	Y	000	100	010	110	001	101	011	111
Ω_Z	Z								
0	00	0	0	0	1	0	1	1	1
1	10	0	0	0	0	0	0	0	0
2	01	0	0	0	0	0	0	0	0
3	11	1	1	1	0	1	0	0	0
		$\varphi(Y) = \varphi(x_2, x_3, x_4)$							
		0	0	0	1	0	1	1	1

z\\φ	0	1
00	0	1
10	0	0
01	0	0
11	1	0

φ – Component of decomposition

Φ - Image of decomposition

Ashenhurst's Fundamental Theorem of Functional Decomposition

The simple disjoint decomposition exists if and only if the corresponding decomposition chart has

at most two distinct column patterns.

Ω_Y	0	.....	N_Y	....	2^k-1
Ω_Z					
0	$f_1(0)$	.....	$f_2(0)$	.....	$f_1(0)$
1	$f_1(1)$	.....	$f_2(1)$	.....	$f_1(1)$
.....	.....	.....	.....	.....	.....
N_z	$f_1(N_z)$	.....	$f_2(N_z)$	.....	$f_1(N_z)$
.....	.....	.....	.....	.....	.....
$2^{n-k}-2$	$f_1(2^{n-k}-2)$	.....	$f_2(2^{n-k}-2)$	.....	$f_1(2^{n-k}-2)$
$2^{n-k}-1$	$f_1(2^{n-k}-1)$	.....	$f_2(2^{n-k}-1)$	.....	$f_1(2^{n-k}-1)$
$\varphi(Y)$	***1***	.....	***0***	.....	***1***

Example

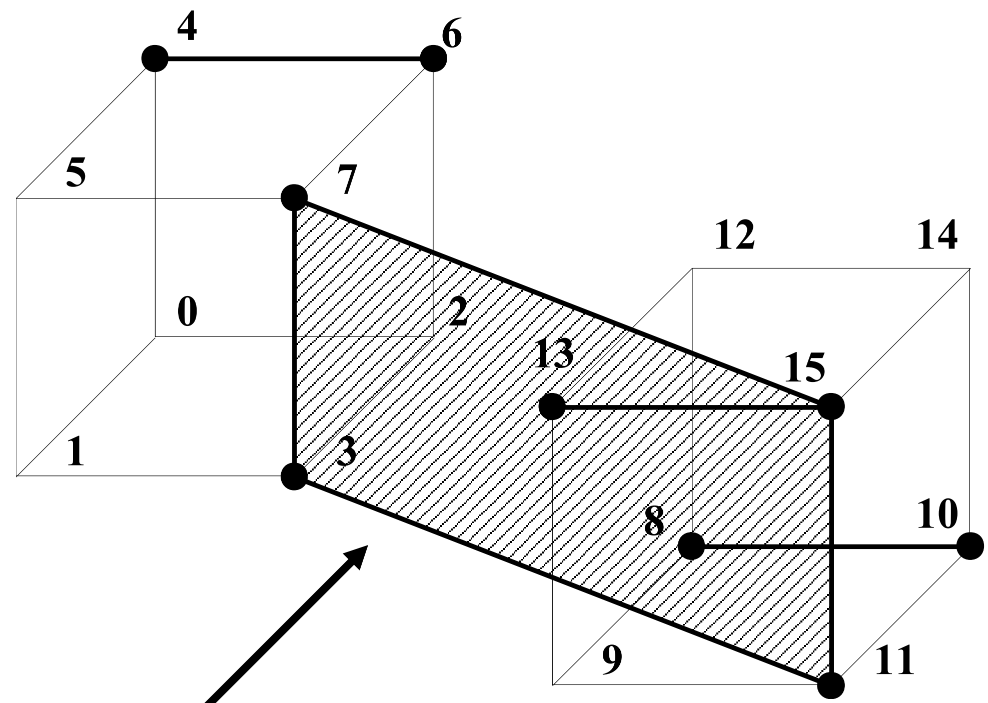
Function of 6 variables: $\boldsymbol{Y = \{x_3, x_4, x_5\}; Z=\{x_0,x_1,x_2\}}$

$T = \{3,4,6,7,8,10,11,13,15,16,18,19,21,23,27,28,30,31,$
$32,34,35,37,39,40,42,43,45,47,48,50.51,53,55,59,60,62,63\}$

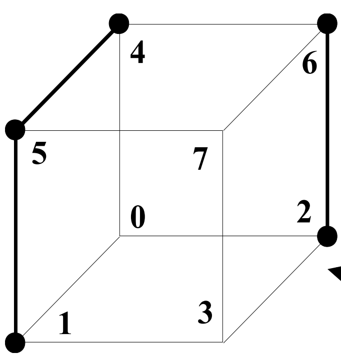
Ω_y	0	1	2	3	4	5	6	7
Ω_z								
0	0	1	1	0	1	1	1	0
1	0	0	0	0	0	0	0	0
2	0	1	1	0	1	1	1	0
3	1	1	1	1	1	1	1	1
4	1	0	0	1	0	0	0	1
5	0	1	1	0	1	1	1	0
6	1	0	0	1	0	0	0	1
7	1	1	1	1	1	1	1	1

$$T_{\varphi(Y)} = \{1,2,4,5,6\}; T_{f_1(Z)} = \{0,2,3,5,7\}; T_{f_2(Z)} = \{3,4,6,7\}.$$

$\varphi(Y)$	0	1
Ω_z		
0	0	1
1	0	0
2	0	1
3	1	1
4	1	0
5	0	1
6	1	0
7	1	1



$$\Phi(Z,\varphi(Y)) = \bar{x}_0 x_2 \bar{\varphi} + x_0 x_1 + x_0 x_2 \varphi + \bar{x}_0 \bar{x}_2 \varphi$$



Component of decomposition:

$$\varphi(Y) = x_3 \bar{x}_4 + \bar{x}_3 x_4 + \bar{x}_4 x_5$$

Body problems

Example 2

Threshold Logic

Threshold Logic

Function $F(x_0, x_1, \ldots., x_{n-1})$ is called threshold function,

if it can be represented as :

$$F(x_0, x_1, \ldots., x_{n-1}) = Sign(\alpha_0 x_0 + \alpha_1 x_1 + \ldots. + \alpha_{n-1} x_{n-1} - \eta) =$$

$$= Sign(\sum_{\mathrm{j}=0}^{\mathrm{n}-1} \alpha_{\mathrm{j}} x_j - \eta),$$

where

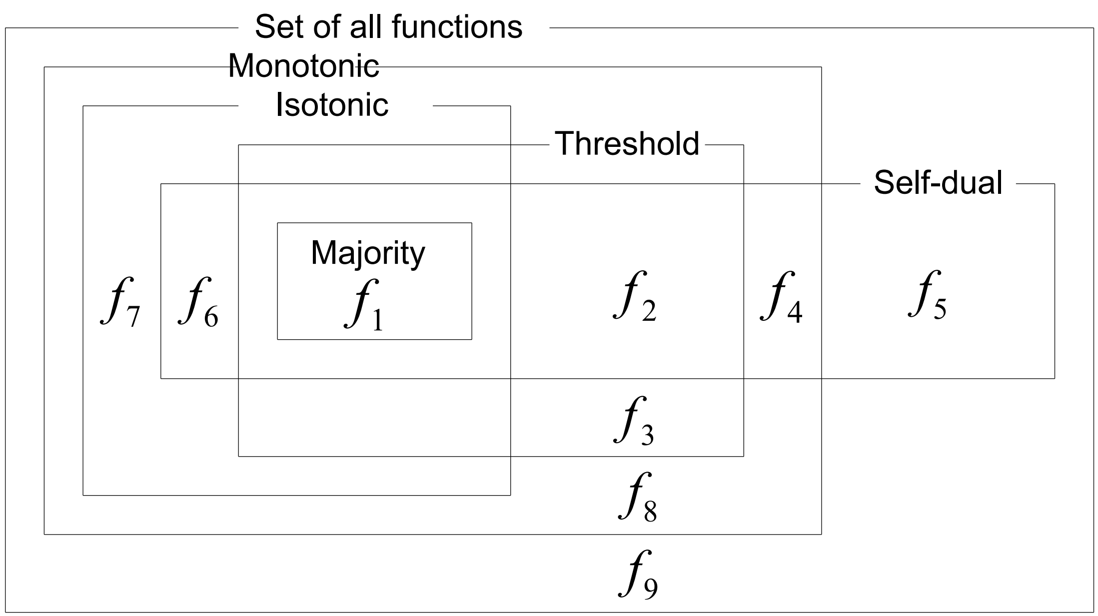
$$Sign\mathrm{A} = \begin{cases} 1 & \text{if} \quad \mathrm{A} \geq 0 \\ 0 & \text{if} \quad \mathrm{A} < 0 \end{cases}$$

α_j - weight of input (variable) x_{j};

η - threshold.

Example: There are 16 functions of two variables, Among them, only two functions are not threshold functions: $x_1 \oplus x_2$ and $x_1 \oplus \overline{x_2}$ All other functions are threshold functions.

Example: All the monotone increasing function with up to three variables are threshold functions. However, the four-variable function $x_1 x_2 \vee x_3 x_4$ is not a threshold function.



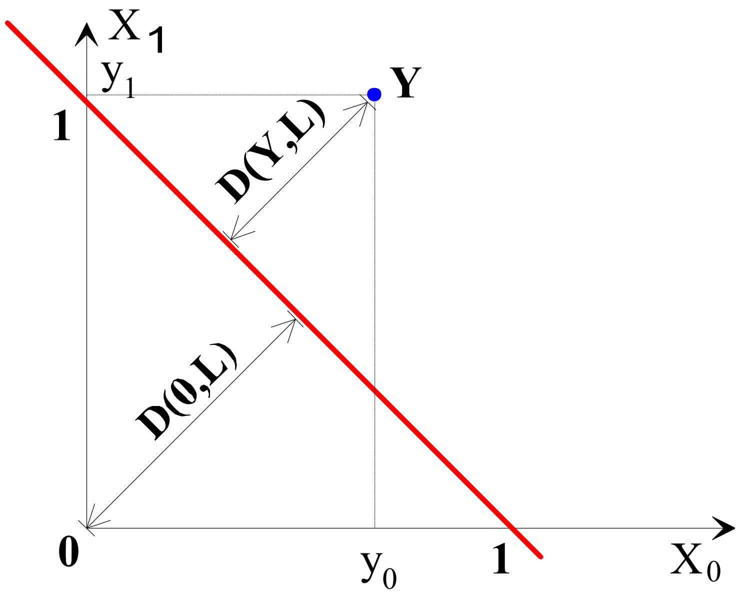
Example of three-inputs majority (voting) function

X_0	X_1	X_2	$X_0+X_1+X_2-2$	$Sign(X_0+X_1+X_2-2)$
0	**0**	**0**	**-2**	**0**
1	**0**	**0**	**-1**	**0**
0	**1**	**0**	**-1**	**0**
1	**1**	**0**	**0**	**1**
0	**0**	**1**	**-1**	**0**
1	**0**	**1**	**0**	**1**
0	**1**	**1**	**0**	**1**
1	**1**	**1**	**1**	**1**

Hyper-geometrical representation of threshold function

Equation $\sum_{j=0}^{n-1} \alpha_j x_j - \eta = 0$ is the equation of a hyper-plane in the n-dimensional space.

For 2-dimentions: $\alpha_0 x_0 + \alpha_1 x_1 - \eta = 0$ equation of a line.



$$x_1 + x_2 - 1 = 0;$$

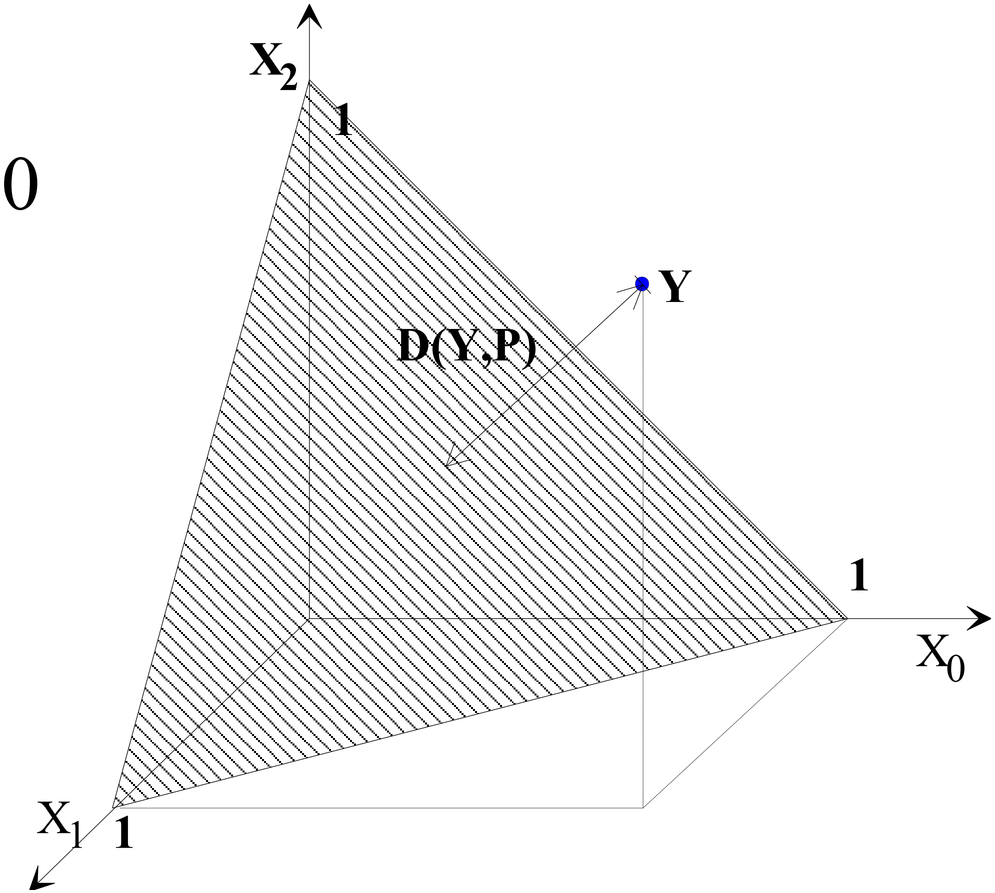
$$D(Y,L) = \frac{\alpha_0 y_0 + \alpha_1 y_1 - \eta}{\sqrt{\alpha_0^2 + \alpha_1^2}} = \frac{y_0 + y_1 - 1}{\sqrt{2}};$$

$$D(0,L) = \frac{-\eta}{\sqrt{\alpha_0^2 + \alpha_1^2}} = \frac{1}{\sqrt{2}};$$

For 3-dimentions:

$$\alpha_0 x_0 + \alpha_1 x_1 + \alpha_2 x_2 - \eta = 0$$

- equation of a plane.



$$x_0 + x_1 + x_2 - 1 = 0;$$

$$D(Y,P) = \frac{\alpha_0 y_0 + \alpha_1 y_1 + \alpha_2 y_2 - \eta}{\sqrt{\alpha_0^2 + \alpha_1^2 + \alpha_2^2}} = \frac{y_0 + y_1 + y_2 - 1}{\sqrt{3}};$$

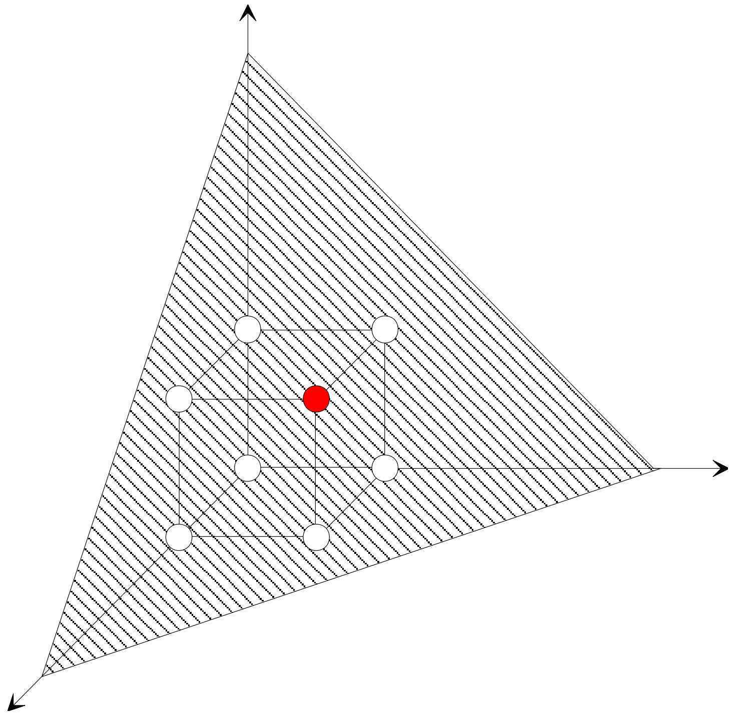
For n-dimensions: $\sum_{j=0}^{n-1} \alpha_j x_j - \eta = 0$ equation of a hyper-plane in the n-dimensional space.

$$D(Y, P^n) = \frac{\sum_{j=0}^{n-1} \alpha_j y_j - \eta}{\sqrt{\sum_{j=0}^{n-1} \alpha_j^2}};$$

As $\sqrt{\sum_{j=0}^{n-1} \alpha_j^2} > 0$, then $Sign\{\mathrm{D}(\mathrm{Y}, \mathrm{P}^{\mathrm{n}})\} = Sign(\sum_{j=0}^{n-1} \alpha_j y_j - \eta)$

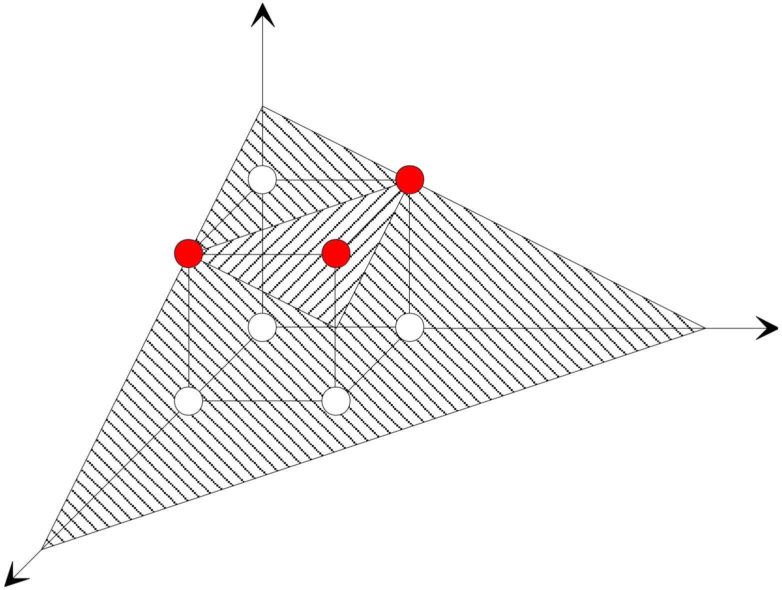
and the value of the threshold function in the point Y (cube vertex) is determined by the sign of the distance from this point to the hyper-plane determined by the weights of the variables and by the threshold.

At all the cube vertices located above or within the hyper-plane, the value of threshold function is equal to 1.
At all the cube vertices located below the hyper-plane, the value of threshold function is equal to 0.

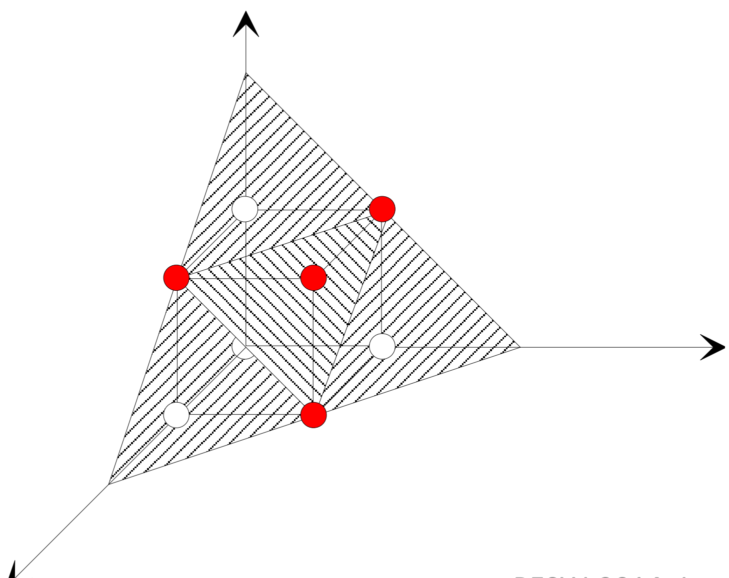


$$F(x_0,x_1,x_2) = x_0 x_1 x_2 =$$

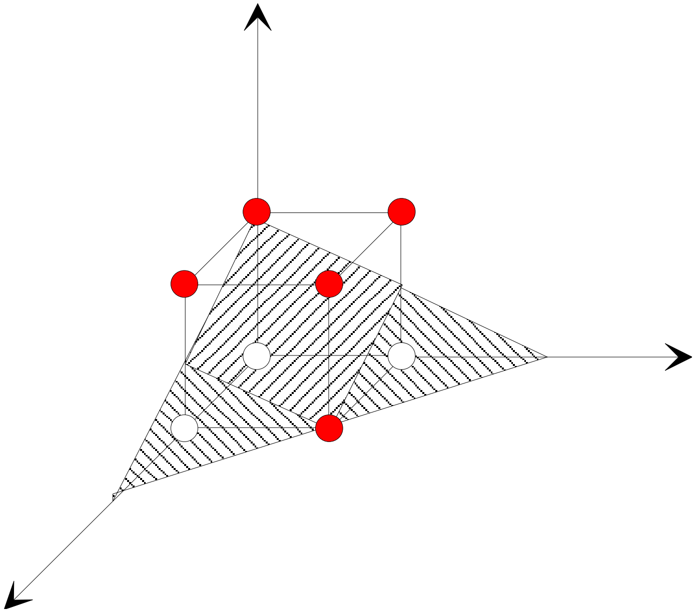
$$= Sign(x_0 + x_1 + x_2 - 3);$$



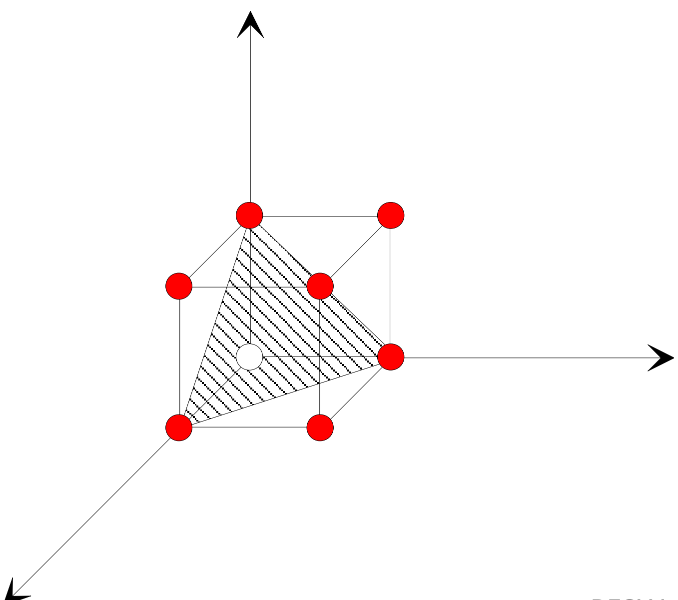
$$F(x_0,x_1,x_2) = x_0x_2 + x_1x_2 =$$
$$= Sign(x_0 + x_1 + 2x_2 - 3)$$



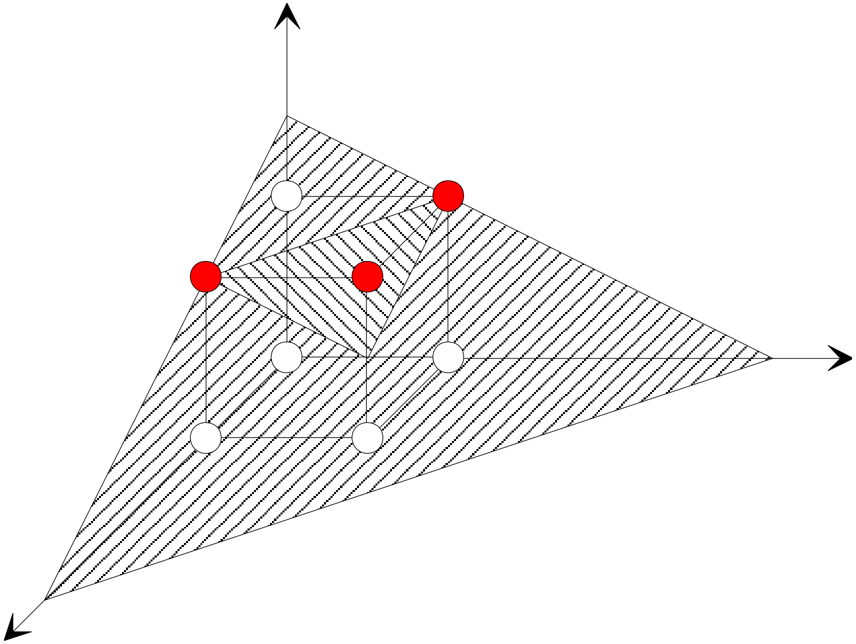
$$F(x_0,x_1,x_2) =$$
$$= x_0x_1 + x_0x_2 + x_1x_2 =$$
$$= Sign(x_0 + x_1 + x_2 - 2);$$

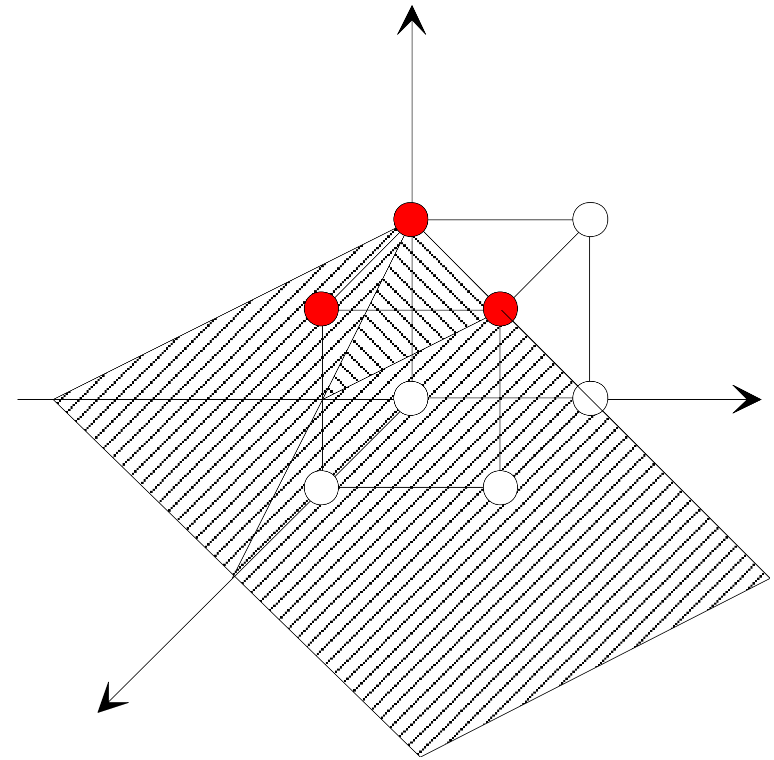


$$F(x_0,x_1,x_2) = x_0 x_1 + x_2 =$$
$$= Sign(x_0 + x_1 + 2x_2 - 2);$$



$$F(x_0,x_1,x_2) = x_0 + x_1 + x_2 =$$
$$= Sign(x_0 + x_1 + x_2 - 1);$$





$$F(x_0,x_1,x_2) = x_0x_2 + x_1x_2 =$$
$$= Sign(x_0 + x_1 + 2x_2 - 3);$$

$$F(x_0,x_1,x_2) = x_0x_2 + \bar{x}_1x_2 =$$
$$= Sign(x_0 - x_1 + 2x_2 - 2);$$

For Boolean function defined by two sets of vertices T and F.

$\Phi(X \in T)=1; \quad \Phi(X \in F)=0.$

The input weights and threshold of the corresponding threshold function are found by solving the task:

$$\min\left(\sum_{j=0}^{n-1} |\alpha_j| + |\eta|\right);$$

$$\sum_{j=0}^{n-1} \alpha_j x_j - \eta \geq 0 \qquad \forall X \in T;$$

$$\sum_{j=0}^{n-1} \alpha_j x_j - \eta < 0 \qquad \forall X \in F;$$

This task is called the standard task of ***linear programming***.

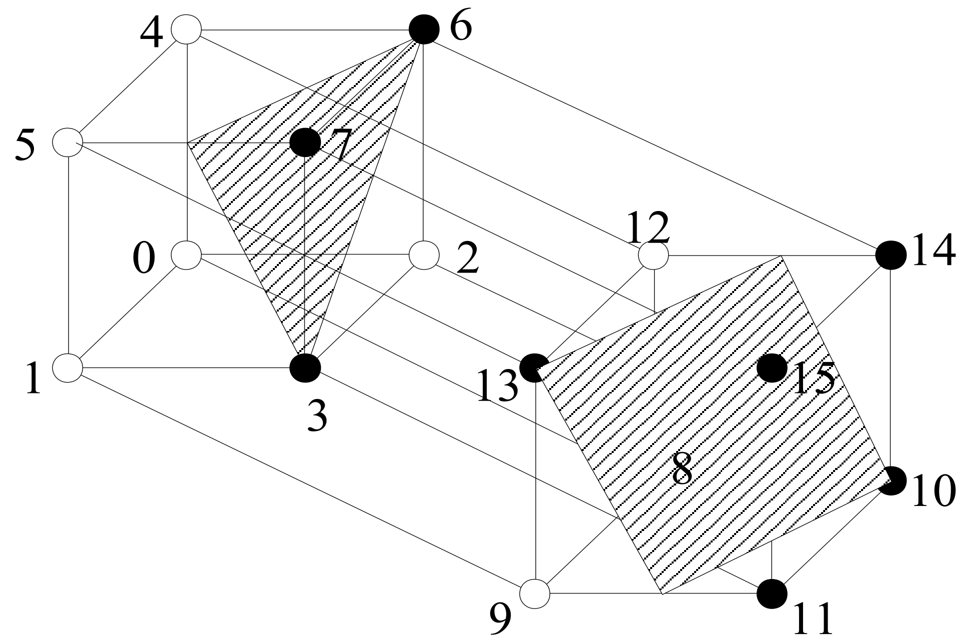
If a solution of this task exists, then for $x_j = \{0,1\}$ it exists in integer numbers.

If a solution doesn't exist, this Boolean function is not a threshold function.

Since a threshold function is monotonous, it is represented by a star of basic elements. *Basic* sets T_0 and F_0 of threshold function are the sets of vertexes, which are maximal diagonal vertices in the basic elements that form stars for the function itself and for its negation.

Example:

Star top is 15 vertex.
T = {3,6,7,10,11,13,14,15}
T_0={3,6,10,13}
Star top of negation function is 0 vertex.
F = {0,1,4,2,5,8,9,12}
F_0={2,5,9,12}



$$\Phi(x_0, x_1, x_2, x_3) = x_0x_1 + x_1x_2 + x_1x_3 + x_0x_2x_3 =$$
$$= Sign(x_0 + 2x_1 + x_2 + x_3 - 3)$$

When synthesizing a threshold element, it is sufficient to solve the linear programming task only for the *basic* sets because, if the inequalities are true for the supporting sets, they are certainly true for all the remaining vertexes.

Branch problems

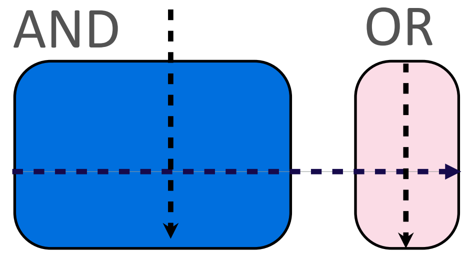
Applied engineering studies caused by and connected with emerging industrial challenges

Branch problems

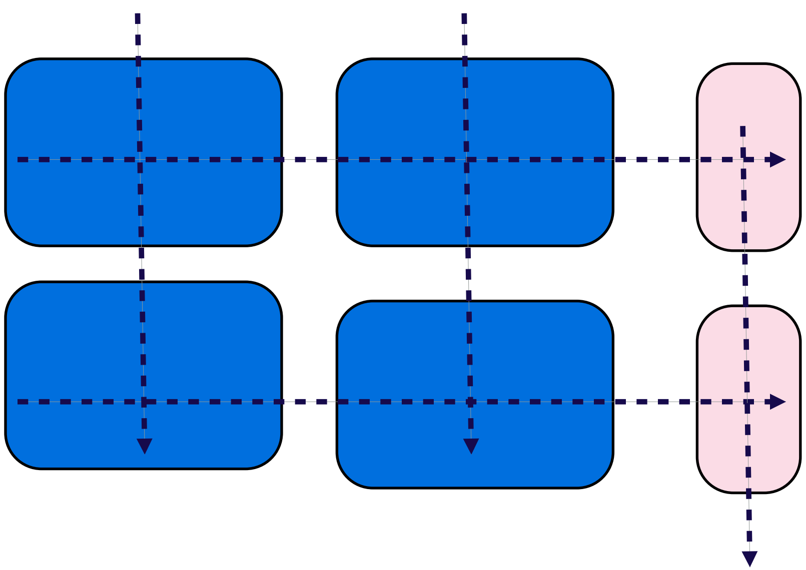
Example 1

DESIGNING FAULT TOLERANT FSM BY NANO-PLA

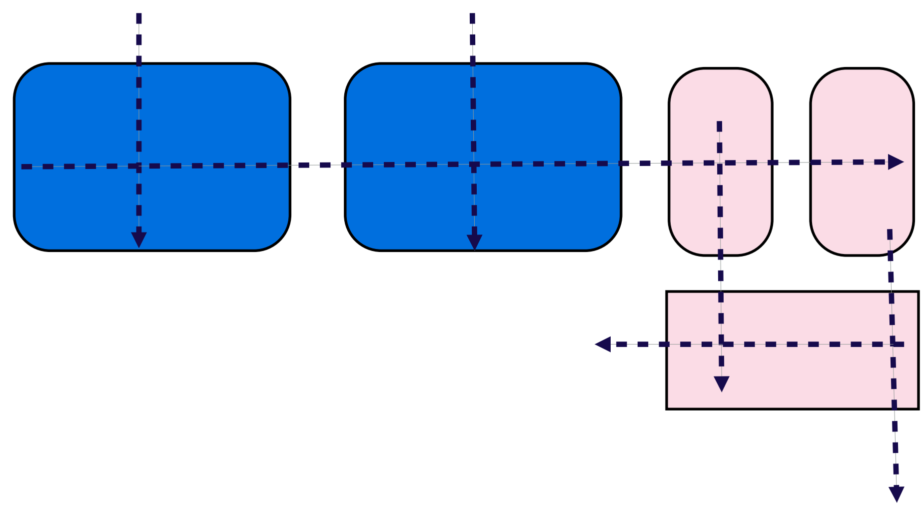
MASKING SCHEMES OF FAULT TOLERANT PLA



Standard PLA

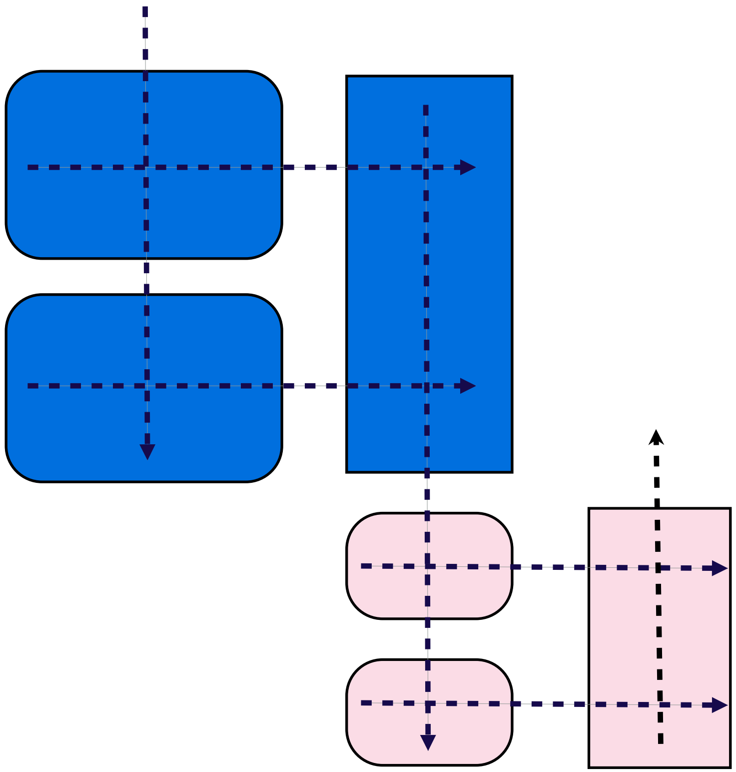


A-O fault masking

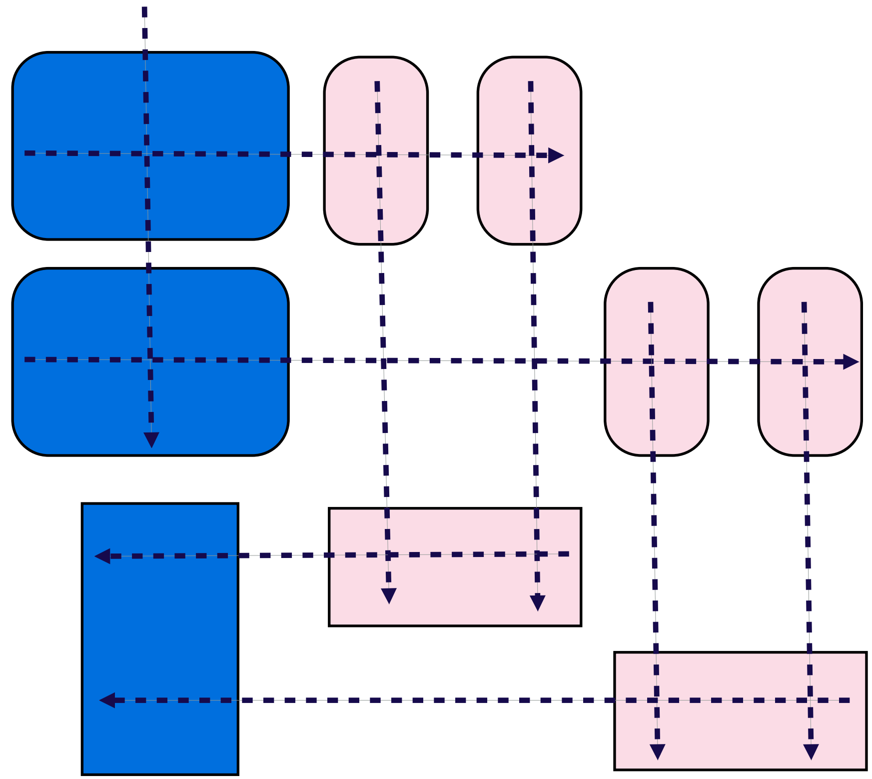


A-O-O fault masking

MASKING SCHEMES OF FAULT TOLERANT PLA



A-A-O-O fault masking



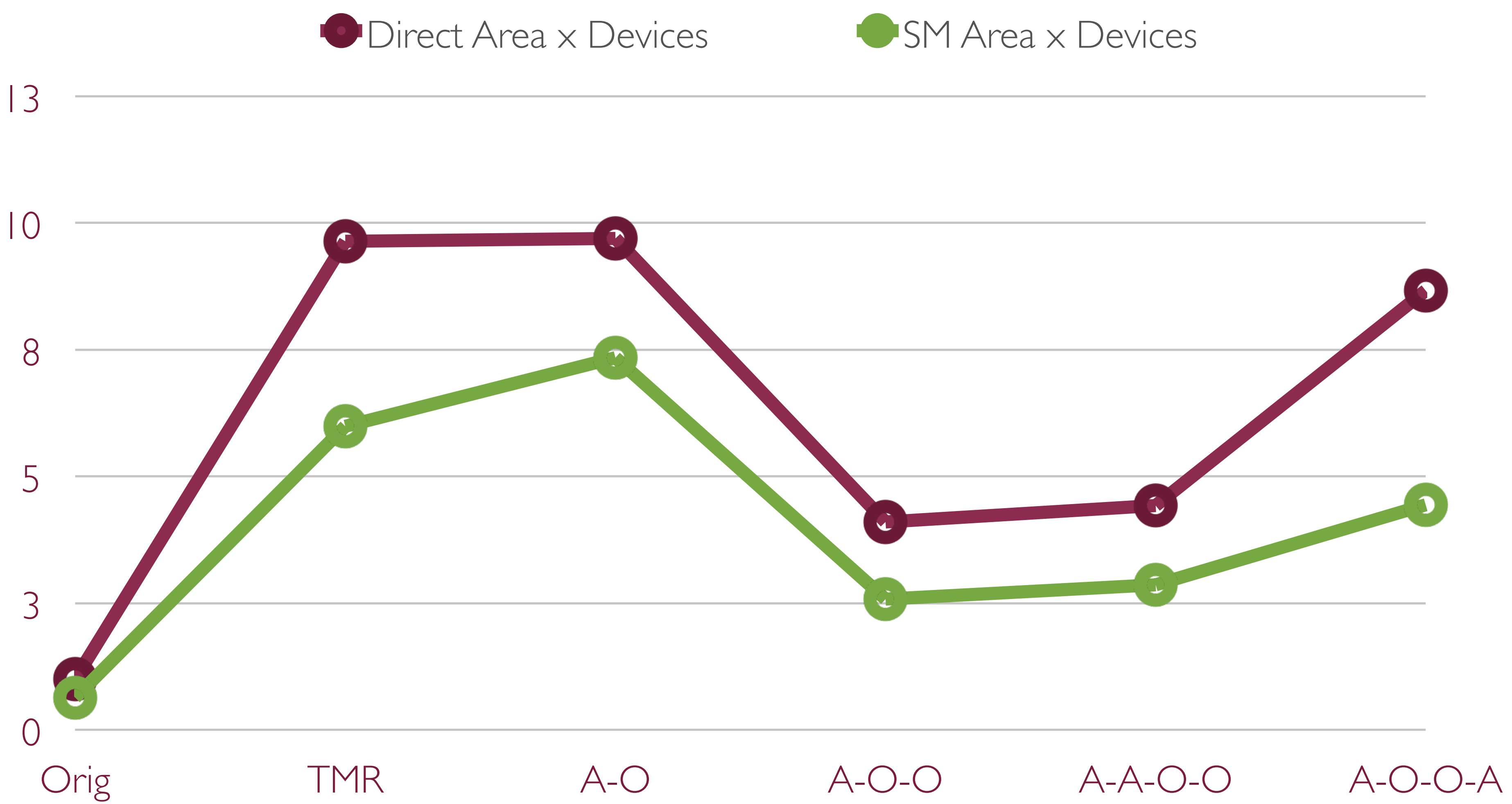
A-O-O-A fault masking

ESTIMATION OF AREA AND DEVICES NUMBERS

$$S_{org} = IP + OP;$$
$$S_{TMR} = 3IP + 3OP + 9P;$$
$$S_{AO} = 4IP + 2OP;$$
$$S_{AOO} = 2IP + 2OP + 2O;$$
$$S_{AAOO} = 2IP + 2OP + 2P + 20;$$
$$S_{AOOA} = 2IP + 4OP + 6O.$$

$$D_{org} = D_A + D_O;$$
$$D_{TMR} = 3D_A + 3D_O + 9P;$$
$$D_{AO} = 4D_A + 2D_O;$$
$$D_{AOO} = 2D_A + 2D_O + 2O;$$
$$D_{AAOO} = 2D_A + 2D_O + 2P + 20;$$
$$D_{AOOA} = 2D_A + 4D_O + 6O.$$

INTEGRAL EFFICIENCY FACTOR
DIRECT vs. SM SCHEME



ALL THE KNOWN METHODS:

- Are based on doubling rows and/or columns of PLA plans.
- Double empty crosspoints of the PLA, which results in unreasonable overhead.
- Don’t use the density parameter to reduce the resulting area.

The approach is oriented to combinational parts of FSMs corresponding to sparse PLAs:

- Is based on decomposition of the initial PLA into a chain of high-density component PLAs.
- Allows transforming the high-density components into a certain fault tolerant form, without excessive doubling of PLA's empty crosspoints.

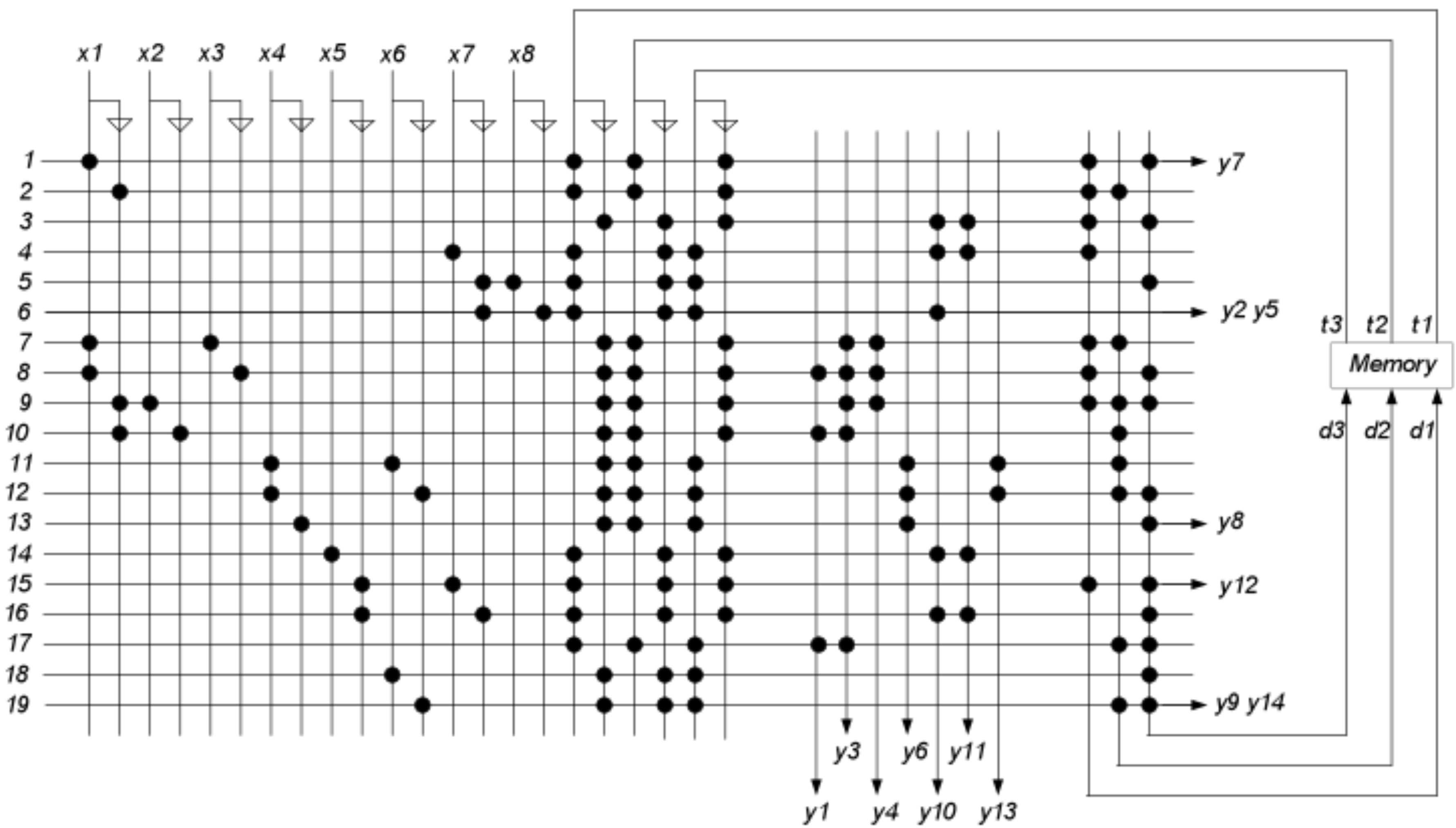
FINITE STATE MACHINES

Synthesis of Fault Tolerant FSM by PLA

Structural Table of FSM

a_m	K(am)	a_s	$K(a_s)$	$X(a_m,a_s)$	$Y(a_m,a_s)$	Y_t	$D(a_m,a_s)$	H
a_1	110	a_3	101	x_1	y_7	Y_8	d_1d_3	1
		a_1	110	$\sim x_1$	–	Y_0	d_1d_2	2
a_2	000	a_3	101	1	$y_{10}y_{11}$	Y_4	d_1d_3	3
a_3	101	a_6	100	x_7	$y_{10}y_{11}$	Y_4	d_1	4
		a_8	001	$\sim x_7x_8$	–	Y_0	d_3	5
		a_2	000	$\sim x_7\sim x_8$	$y_2y_5\ y_{10}$	Y_1	–	6
a_4	010	a_1	110	$x_1\ x_3$	y_3y_4	Y_2	d_1d_2	7
		a_3	101	$x_1\sim x_3$	$y_1y_3y_4$	Y_3	d_1d_3	8
		a_7	111	$\sim x_1x_2$	y_3y_4	Y_2	$d_1d_2d_3$	9
		a_4	010	$\sim x_1\sim x_2$	y_1y_3	Y_6	d_2	10
a_5	011	a_4	010	x_4x_6	y_6y_{13}	Y_9	d_2	11
		a_5	011	$x_4\sim x_6$	y_6y_{13}	Y_9	d_2d_3	12
		a_8	001	$\sim x_4$	y_6y_8	Y_5	d_3	13
a_6	100	a_2	000	x_5	$y_{10}y_{11}$	Y_4	–	14
		a_3	101	$\sim x_5x_7$	y_{12}	Y_{10}	d_1d_3	15
		a_8	001	$\sim x_5\sim x_7$	$y_{10}y_{11}$	Y_4	d_3	16
a_7	111	a_5	011	1	y_1y_3	Y_6	d_2d_3	17
a_8	001	a_8	001	x_6	–	Y_0	d_3	18
		a_5	011	$\sim x_6$	y_9y_{14}	Y_7	d_2d_3	19

PLA IMPLEMENTATION OF FSM



INITIAL FSM

The combinational part of the initial FSM implements transformation

$(X \cup T) \Rightarrow (Y \cup D)$;

$X = \{x_1, \ldots, x_L\}$ - input variables $X = X_1 \cup X_2$, $X_1 \cap X_2 = \varnothing$;

$Y = \{y_1, \ldots, y_N\}$ - output variables $Y = \Upsilon_1 \cup \Upsilon_2$, $\Upsilon_1 \cap \Upsilon_2 = \varnothing$;

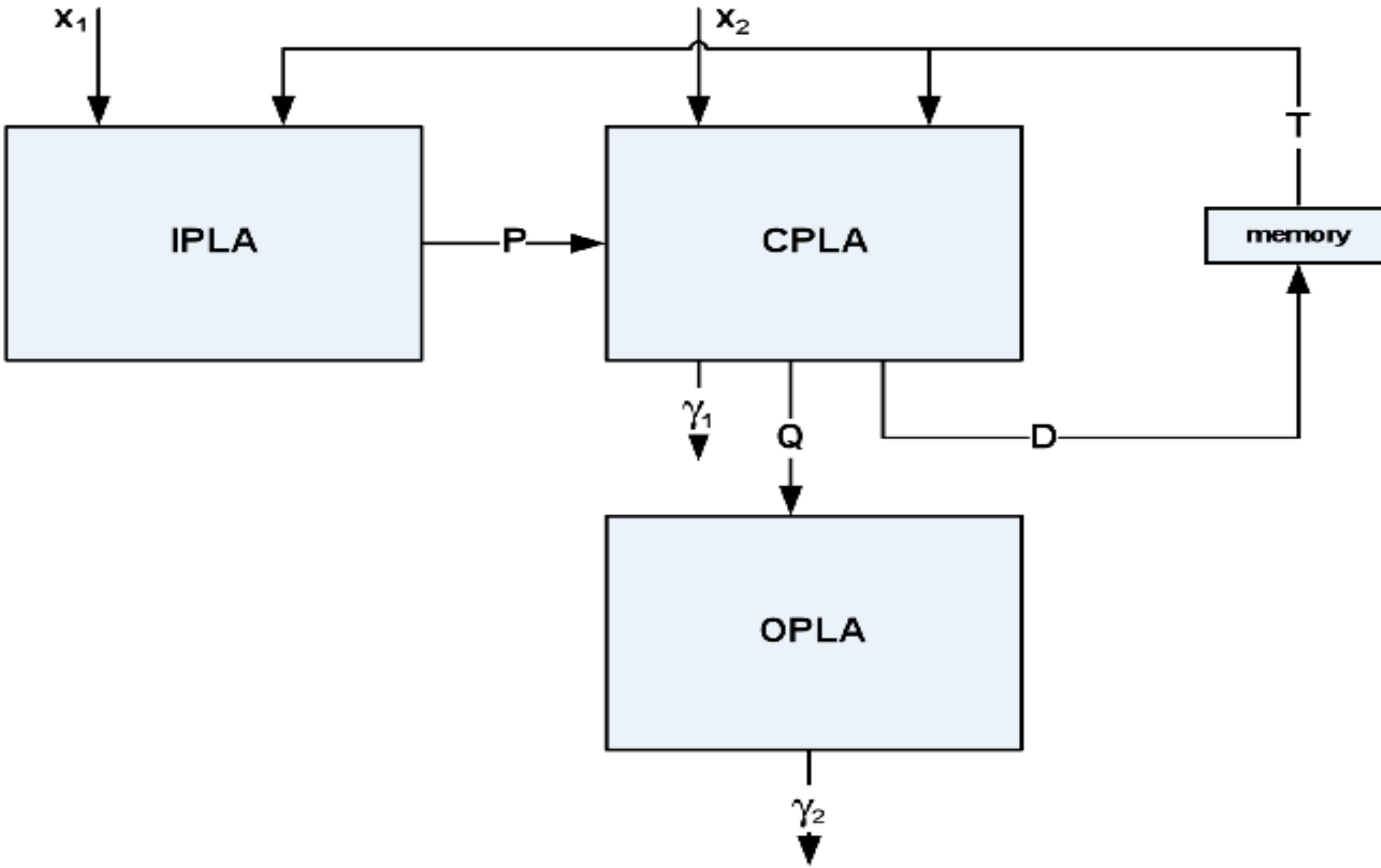
$T = \{t_1, \ldots, t_R\}$ - present state variables;

$D = \{d_1, \ldots, d_R\}$ - next state variables.

SIX MATRIX (SM) FSM ARCHITECTURE

PLA implementation

SM ARCHITECTURE



SM TRANSFORMATIONS

Two main transformations form the SM architecture :

- transformation of input variables
- transformation of output variables.

Both of the transformations are implemented by corresponding PLAs.

The SM architecture comprises three PLAs :

inputs transformation PLA (Input PLA): $\left(X_1 \cup T\right) \Rightarrow P$,

core transformation PLA (Core PLA): $\left(X_2 \cup P \cup T\right) \Rightarrow \left(Q \cup \Upsilon_1 \cup D\right)$,

outputs transformation PLA (Output PLA): $Q \Rightarrow \Upsilon_2$.

Two additional sets of auxiliary variables are introduced into the SM architecture :

$P = \left\{p_1, \ldots, p_K\right\}$- output variables of Input PLA;

$Q = \left\{q_1, \ldots, q_M\right\}$- output variables of Core PLA.

INPUTS TRANSFORMATION PLA

$X(a_i)$ - set of input variables that determine the transitions from state a_i.

$$X(a_1)=\{x_1\};\ X(a_5)=\{x_4,x_6\};\ X(a_2)=\varnothing;$$
$$X(a_6)=\{x_5,x_7\};\ X(a_3)=\{x_7,x_8\};\ X(a_7)=\varnothing;$$
$$X(a_4)=\{x_1,x_2,x_3\};\ X(a_8)=\{x_6\}.$$

Transformation of the original input variables - replacing the set of variables X with a set of new variables P. $|P|\leq 3$.

INPUTS TRANSFORMATION

To optimize the PLA implementation of the inputs transformation PLA, a specific state assignment has to be applied :

$$K(a_1) = 110; K(a_2) = 000; K(a_3) = 101;$$
$$K(a_4) = 010; K(a_5) = 011; K(a_6) = 100;$$
$$K(a_7) = 111; K(a_8) = 001$$

REPLACEMENT OF INPUT VARIABLES

a_m	p_1	p_2	p_3
a_1	x_1	-	-
a_2	-	-	-
a_3	x_7	x_8	-
a_4	x_1	x_2	x_3
a_5	x_4	x_6	-
a_6	x_7	x_5	-
a_7	-	-	-
a_8	-	x_6	-

After minimization :

$$p_1 = t_2 \bar{t}_3 x_1 + \bar{t}_2 x_7 + t_2 t_3 x_4;$$

$$p_2 = t_1 t_3 x_8 + t_2 \bar{t}_3 x_2 + \bar{t}_1 t_3 x_6 + \bar{t}_2 \bar{t}_3 x_5$$

OUTPUTS TRANSFORMATION

The outputs transformation PLA transforms a set Q to a subset Υ_2

$$Y_0 = \varnothing;\ Y_1 = \{y_2, y_5, y_{10}\}; Y_2 = \{y_3, y_4\}; Y_3 = \{y_1, y_3, y_4\};$$

$$Y_4 = \{y_{10}, y_{11}\}; Y_5 = \{y_6, y_8\}; Y_6 = \{y_1, y_3\};$$

$$Y_7 = \{y_9, y_{14}\}; Y_8 = \{y_7\}; Y_9 = \{y_6, y_{13}\}; Y_{10} = \{y_{12}\}.$$

Each Y_i - set may be associated with a binary code $K(Y_j)$.

B_j - a minterm of variables $q_1, \ldots, q_M$ corresponding to $K(Y_j)$.

$$y_1 = B_3 + B_6;\ y_2 = B_1;\ y_3 = B_2 + B_3 + B_6;$$

$$y_4 = B_2 + B_3;\ y_5 = B_1;\ y_6 = B_5 + B_9;\ y_7 = B_8;$$

$$y_8 = B_5;\ y_9 = B_7;\ y_{10} = B_1 + B_4;\ y_{11} = B_4;$$

$$y_{12} = B_{10};\ y_{13} = B_9;\ y_{14} = B_7.$$

CORE TRANSFORMATION

The core transformation PLA implements the transformation :
$\left(X_2 \cup P \cup T\right) \Rightarrow \left(Q \cup Y_1 \cup D\right)$.

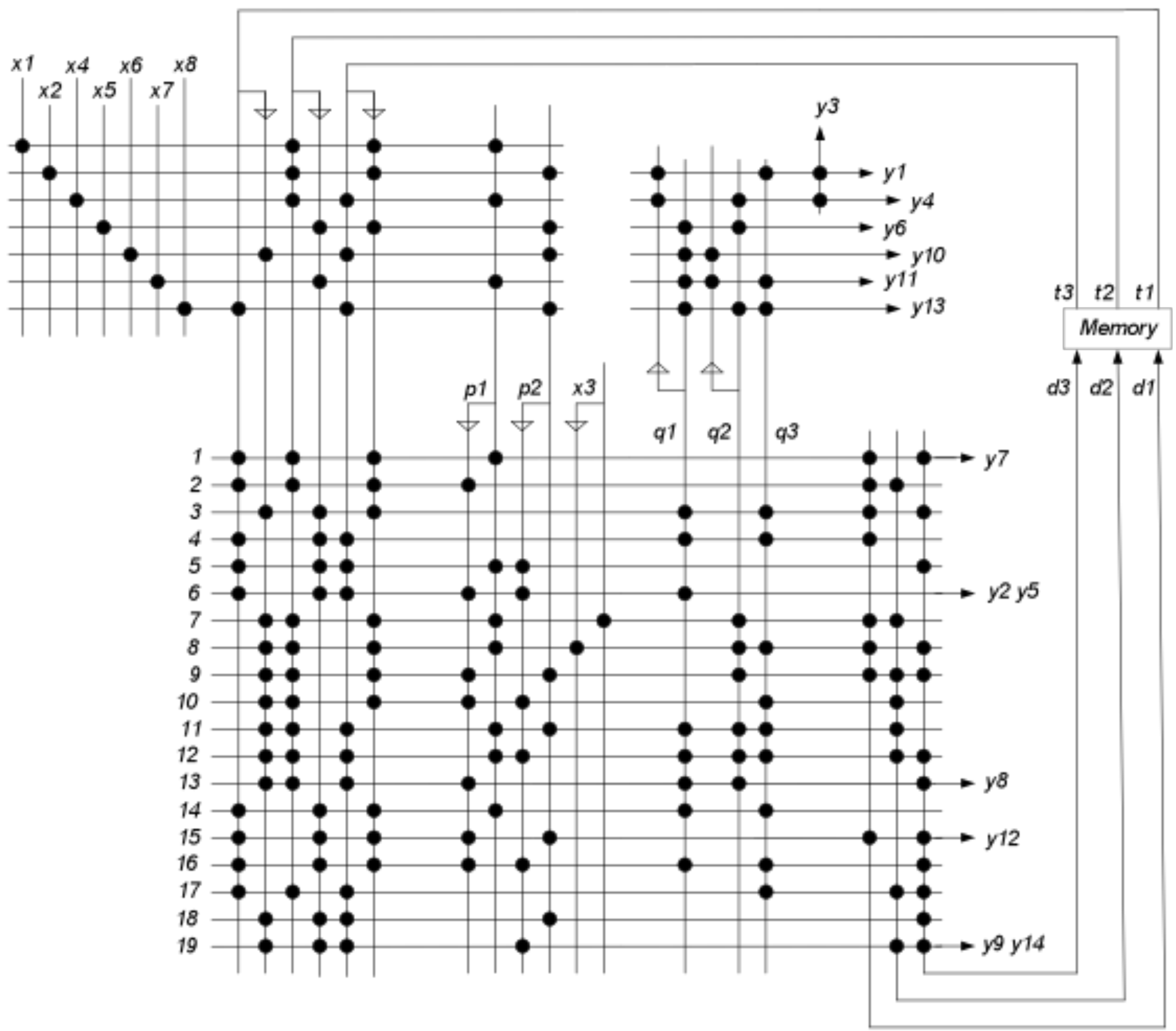
The Core FSM differs from the initial FSM by its inputs and outputs.
It uses new P variables instead of input variables X_1, and new Q variables instead of output variables Y_2.

The transition functions and the next state functions of the core FSM remain the same as in the initial FSM.

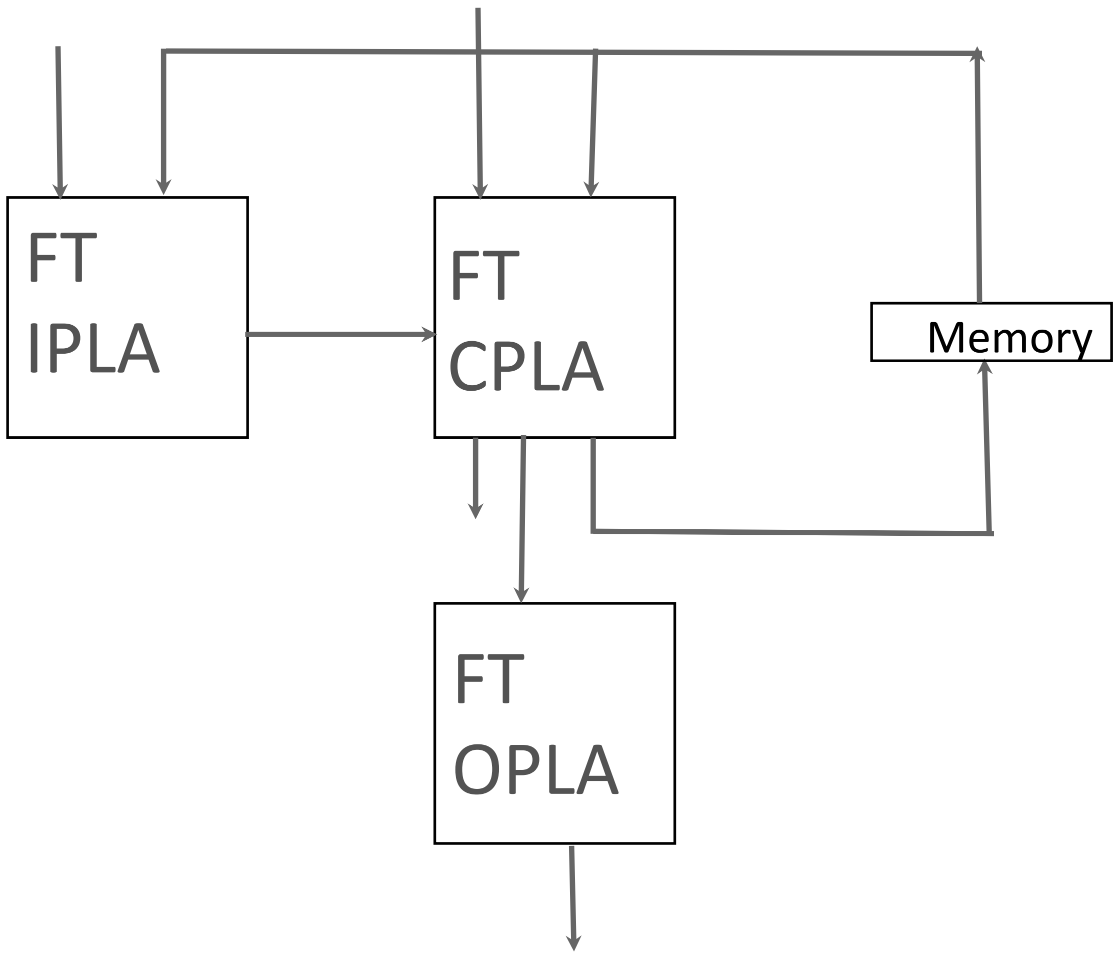
CORE FSM

a_m	K(am)	a_s	$K(a_s)$	$P(a_m,a_s)$	$Q(a_m,a_s)$	$D(a_m,a_s)$	H
a_1	110	a_3	101	p_1	B_8	d_1d_3	1
		a_1	110	$\sim p_1$	B_0	d_1d_2	2
a_2	000	a_3	101	1	B_4	d_1d_3	3
a_3	101	a_6	100	p_2	B_4	d_1	4
		a_8	001	$\sim p_1p_2$	B_0	d_3	5
		a_2	000	$\sim p_1\sim p_2$	B_1	–	6
a_4	010	a_1	110	$p_1\ x_3$	B_2	d_1d_2	7
		a_3	101	$p_1\sim x_3$	B_3	d_1d_3	8
		a_7	111	$\sim p_1p_2$	B_2	$d_1d_2d_3$	9
		a_4	010	$\sim p_1\sim p_2$	B_6	d_2	10
a_5	011	a_4	010	p_1p_2	B_9	d_2	11
		a_5	011	$p_1\sim p_2$	B_9	d_2d_3	12
		a_8	001	$\sim p_1$	B_5	d_3	13
a_6	100	a_2	000	p_2	B_4	–	14
		a_3	101	$\sim p_2p_1$	B_{10}	d_1d_3	15
		a_8	001	$\sim p_2\sim p_1$	B_4	d_3	16
a_7	111	a_5	011	1	B_6	d_2d_3	17
a_8	001	a_8	001	p_2	B_0	d_3	18
		a_5	011	$\sim p_2$	B_7	d_2d_3	19

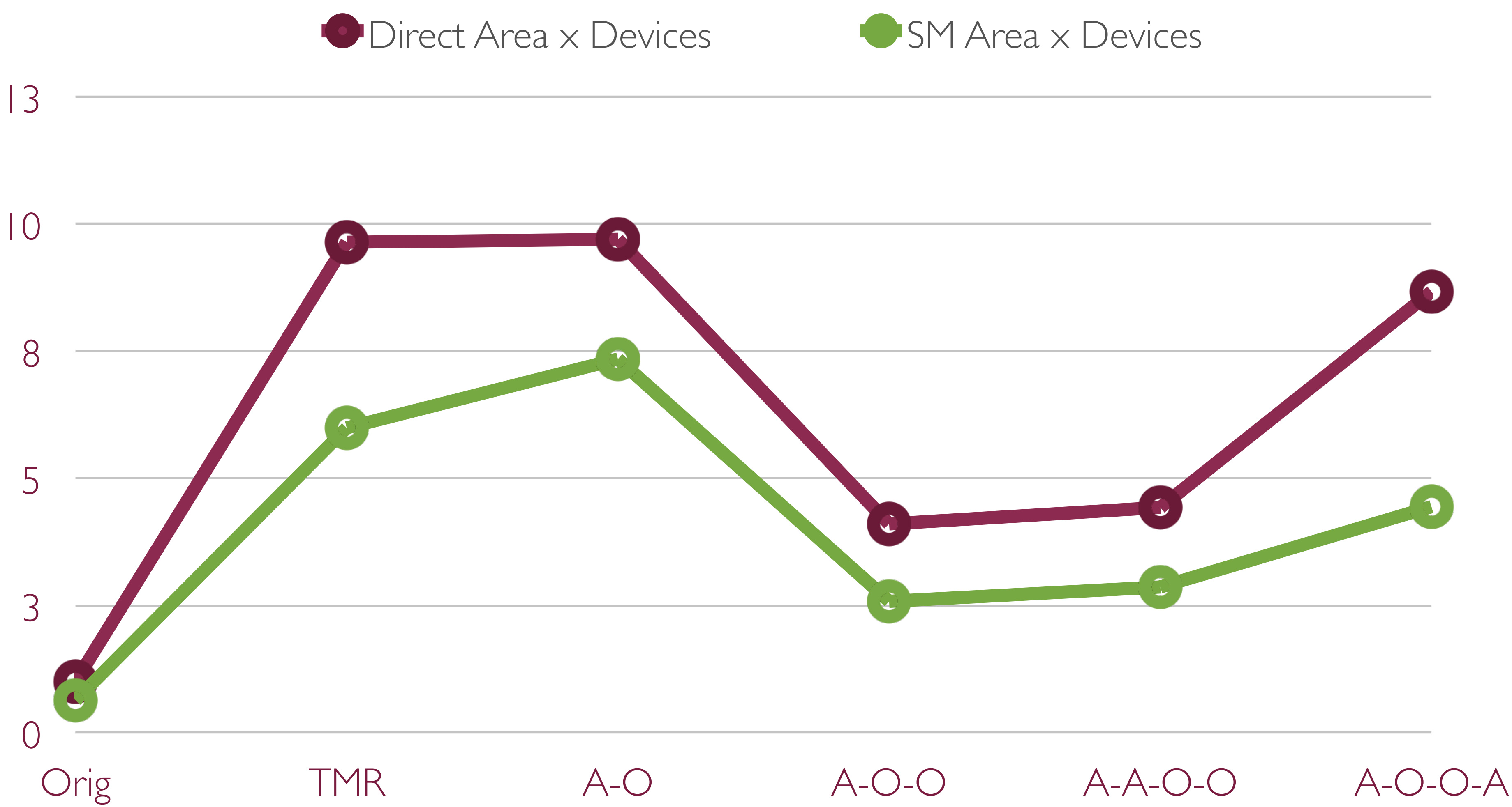
OPTIMIZED SIX MATRIX



FAULT TOLERANT SM ARCHITECTURE



INTEGRAL EFFICIENCY FACTOR DIRECT vs. SM SCHEME



CONCLUSIONS

- A taxonomy of research in Digital Design is presented
- Root, body and branch types of research are considered
- Root type belong to the pure math and forms the Digital Design as an academic discipline
- The body researches comprise fundamental studies of the Digital Design itself, enriching the discipline.
- The branch researches include applied engineering studies caused by and connected with emerging industrial challenges.
- Digital Design is on its mature stage of evolution
- New ideas are extremely important