Properties of Boolean functions in Cognitive Complexity Measure

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***Abstract*— The study focused on three different measures of cognitive complexity: Minimal Description (MD), Structural Complexity (SC) and Mental Model (MM). With respect to these complexity measures, the relationship between symmetry (S), linearity (L) and monotony (M) of Boolean concepts and the different complexity measures presented. Effect of properties of Boolean functions on three different measures of cognitive complexity is studied on solving problems of Boolean recognition and Boolean reconstruction.**

Keywords—Boolean Concepts, Recognition, Reconstruction, Faults, Digital systems

An important issue of the theory of concept learning is the ability to predict the difficulty in learning different types of concepts. Difficulties in learning Boolean concepts have been studied extensively by Shepard, Hovland, and Jenkins (SHJ) [1]. This study focused on Boolean concepts with three binary variables, where the concept receives value “1” for 4 out of 8 possible combinations and value “0” for the remaining 4 combinations. Some of the 70 possible Boolean concepts are congruent (NPN-equivalent). They can be divided into six subcategories. The six SHJ subcategories can be represented graphically as follows (Fig 1).

Results of the SHJ study are significant since SHJ formulated two informal hypotheses. The first hypothesis states that the number of literals in the minimal expression corresponds to the level of concept’s cognitive complexity.

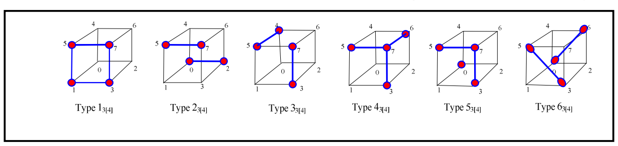

Fig. 1. SHJ category types.

The second hypothesis states that ranking the cognitive complexity among the concepts in each type depends on the number of binary variables in the concept. Feldman [2], based on the conclusions from the SHJ study, defined a quantitative relationship between the level of the cognitive complexity of Boolean concepts. According to [2], the complexity measure of a Boolean concept is the number of literals in the minimal SOP expression that represents the concept. Since there are several minimization techniques, Vigo [3] proposed using the specific Quine-McCluskey (QM) technique to obtain the minimal description (MD). The definition of the Boolean concept’s complexity as a minimal number of literals in a minimal expression has two following drawbacks.

1. Since the complexity is defined as the number of literals in the minimal expression and the expressions can be minimized using different techniques, a single complexity cannot be obtained.

2. Studies show that the Boolean concept “xor” is learned and acquired as a concept by humans to not harder than the Boolean concept “or”.

Aware of the above issues, Vigo [4], developed an alternative approach for calculating the complexity of a Boolean concept by defining a so-called structural complexity (SC). The approach is based on a Boolean derivative. Vigo’s account of the invariance of concepts, as he acknowledges, does not specify how individuals learn concepts. He assumes that cognitive processes could detect invariances by comparing a set of instances to the set yielded by the partial derivative of each variable. Calculations at the foundation of the approach are complex. Mental processes are not taken into account at the foundation of the calculations. SC approach do not comprise a mental representation of concepts or processes.

As an alternative to the complexity theories presented above that predict the difficulty in learning Boolean concepts, a Mental Model (MM) complexity theory [5] is proposed. The MM theory presumes that the mind is not logical and also not a probability system but rather, in essence, it conducts simulations. The theory applies to inclusion thinking and it presumes that when people think, they are attempting to imagine the possibilities of the presumptions that they must address and they draw conclusions. Each of the combinations from all the possibilities that receive a “1” in the result is a MM. When people learn the concepts they can minimize the number of mental models by cancelling irrelevant variables relative to other variables with a known logical value. The number of models of the concept that obtained after minimizing the irrelevant variables predict the difficulty of learning the concept and define the complexity measure of the concept’s degree of difficulty.

The recognition problem is can be modeled by using a visual representation of various objects of a common pattern. Solving the recognition problem may thus be considered as recognizing a visually represented Boolean concept, with further formulation of the concept.

The process of finding and reconstructing operating mechanisms in a given functional system of a digital electronic unit is defined as reconstructing (RE) [6]. RE problem means reconstructing a Boolean function implemented within a given “black box”.

The experiment was conducted in two stages for 13 concepts, where each concept was described by means of a Boolean expression in Table 1. On the first stage, RE problems was examined using a black box that could be used to control the lighting of a bulb using independent switches. During the second stage, recognition problems (Fig 2) were examined using a questionnaire with 13 patterns, where each pattern represents one of the 13 concepts examined, respectively.

Our paper deals with the question: What is the relationship between property of a Boolean concept and the cognitive complexity of the Boolean concepts? This question refers to the research hypothesis that properties of Boolean functions affect the complexity beyond the complexity measures that were presented.

With regards to property of a Boolean concept, all the complexity measures that we relied on failed to predict the difficulty in solving reconstruction problems. Among the symmetric functions that were tested, the “xor” operator was more complex to solve in the two types of problems compared to other symmetric concepts that were examined. Monotonic and symmetrical concepts are the easiest solution. The structural complexity (SC) measures better predictor compared to the minimal description (MD) and Mental Model (MM), except concepts with properties of symmetry, linearity and monotonicity.

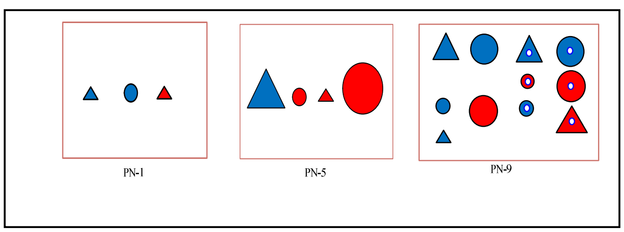

Fig. 2. patterns for CN 1, 5 and 12

Boolean Concept (CN)	MD	SC	MM	Property of Concept
$\bar{b}(a+c)$	3	1.54	2	-
$\bar{a}c+b\bar{c}$	4	2.14	3	-
$\bar{a}\,\bar{b}+ab=\overline{a\oplus b}$	4 (3)	2.14	2	S+L
$a(b+c)+bc$	5	2.14	3	S+M
$(\bar{a}+\bar{b})\bar{c}+abc=\overline{c\oplus(ab)}$	6 (3)	2.34	3	-
$a+bc+\bar{b}\,\bar{c}=a+\overline{b\oplus c}$	5 (3)	2.79	3	-
$a\,b\,\bar{c}+\bar{a}\,b\,c+a\,\bar{b}\,c=b(a\oplus c)+a\,\bar{b}\,c$	9 (6)	3	3	S
$\bar{a}(b+c)+bc$	5	2.14	3	-
$\bar{a}\,b\,d+b\,\bar{c}\,\bar{d}+a\,\bar{b}\,\bar{c}\,d$	10	2.95	3	S+L
$a(\bar{b}\,c+\bar{c}\,b)+\bar{a}(\bar{b}\,\bar{c}+b\,c)=\overline{a\oplus b\oplus c}$	10 (3)	4.00	4	S+L
$a(b\,c+\bar{c}\,\bar{b})+\bar{a}(\bar{b}\,c+b\,\bar{c})=a\oplus b\oplus c$	10 (3)	4.00	4	S+L
$a(b+c+d)+b(d+c)+cd$	9	4.48	6	S+M
$\bar{a}(\bar{b}+\bar{c}+\bar{d})+\bar{b}(\bar{d}+\bar{c})+\bar{c}\,\bar{d}$	9	4.48	6	S+M

Table 1. The 13 concepts were tested during the experiment and their descriptions according to MD minimal descriptions using “xor”, SC, MM and Property of Concept (S), (L) and (M).

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